From a viewpoint essentially oriented towards problems of strength of materials, modern Structural Mechanics considers the balance equations of Statics as fundamental in the formulation of the equilibrium problem.

During the course of history, however, scientific reflections and theories worked out with regard to «mechanical questions» were almost always characterised by their being focused mainly on the analysis of possible or real movements of the systems, or the geometric parameters that characterised them. This circumstance is perhaps connected with the fact that, before the introduction of abstract quantities and concepts through the use of mathematical language, «mechanical questions» were regarded as reflections concerning practical facts of an empirical nature, in the search for the natural law that governed the phenomenon. In particular, the problem of equilibrium has often been analyzed as the search for the «limit condition» at which a mobile system could be placed in a state of non-mobility.

It was this strong emphasis on the «not activated, but possible» mobility of the system which led to the main reference of the scientific theories worked out over the centuries, the so-called «Principle» of Virtual Works, being assumed as a natural law, or a fundamental postulate.

All the «objects» that have constituted the history of Mechanics are mechanisms; all these «objects» have in common the fact that they are mobile systems, for which the condition of equilibrium requires particular values of the forces applied—usually weights. For such systems, the formulation of the equilibrium through the equations of balance required that Statics be considered as a particular case of Dynamics.

On the other hand, once the concept of weight as a being that has the tendency (which is, moreover, empirically demonstrable) to move downwards and is, therefore, susceptible to movement, was accepted, it is not surprising that the guiding principle of scientific thought was PVW: the search for equilibrium as a condition of «non-activated motion» placed the emphasis on kinematic analysis of the system—and therefore on geometry—through the link with weights, in the search for a law in which the science of weights was governed by geometry. The discovery of the particular condition of «non-activated» motion, that separates equilibrium from motion—or, in static terms, from collapse—coincides with what we nowadays call «limit analysis».

It is in this context that some theories of vaulted structures from Leonardo to La Hire were discussed and re-examined in a previous paper (Sinopoli 2002), in the search for an interpretation that was as close as possible to the thought of the time, while not excluding the use of modern formal instruments to enable an easier and more correct reading. It is along the same lines that the analysis and theories on
vaulted structures proposed by Lorenzo Mascheroni in his «Nuove ricerche sull’equilibrio delle volte» (1785) will be discussed and re-examined in this paper. It is with Mascheroni, in fact, that the analysis proposed by La Hire finds its justification, and arch collapse is examined by means of a mature formulation, based on kinematic analysis of the system.

THE ROLE OF MECHANICS AND STATICS IN SOLVING PROBLEMS REGARDING ARCHES AND VAULTS

As a confirmation of the fact that «mechanical questions» were for a long time considered as theoretical reflections on practical questions of a basically empirical kind, in the preface to «Nuove ricerche sull’equilibrio delle volte» (Mascheroni 1785), Mascheroni affirms that both Statics and Mechanics:

... sono arti dirette all’uso, e la complicazione delle molteplici circostanze, che alterando lo stato della questione fanno che la sola speculazione riesca a decisioni lontanissime dall’esperienza, facilmente si vede dovere la teorica aver predominio nella Statica; siccome per lo contrario dovere la pratica essere consultata a preferenza nella Meccanica. Questa seconda (la Meccanica) procurando il moto trova gli intoppi nelle resistenze dei mezzi, nelle asprezze delle superfici, nelle tenacità, ed attrazioni varie delle materie, il qual genere di circostanze non è così facile di perfettamente rilevare per via di principj, e molto meno di assoggettare alla precisione del calcolo. Ma dovendosi pure ad esse avere tutto il riguardo da chi desidera conseguire l’effetto dei suoi tentativi, quindi è avvenuto che Geometri anche sommi hanno fatto precedere diligentissime e per più capi diversificate prove di fatto, dalle quali raccoglier potessero qualche legge da tenersi poi nel luogo d’un principio nella risoluzione de’ Problemi. (Mascheroni 1785, XXV).1

According to Mascheroni, therefore, Mechanics, considered as a science of motion, cannot be subjected to theoretical analyses due to resistance by the media and all the phenomena which are not controllable, but above all, with regard to such phenomena there are no principles to which reference can be made. Nevertheless, precisely because it is science of motion, it is in the geometry of movements that analyses and factual proof can be obtained; that is, theorems concerning kinematics, to be considered as principles in the solution of problems. However, it is the next observation that becomes a key to the interpretation of the role of Geometry in Mechanics and therefore also in Statics. In fact, Mascheroni continues thus:

Al contrario la Statica cercando di introdurre in varj aggregati di corpi equilibrio e quiete trova ajuto al suo scopo in quelle medesime cose che contrastano colla Meccanica; sicché qualora essa abbia colla scorta della Geometria trovato il sito, e la distribuzione di quelle materie, che deggion star ferme, non può per conto delle circostanze annoverate di sopra se non istarsene più sicura. (Mascheroni 1785, XXV)

Statics, which is concerned not with movement but with a search for conditions of equilibrium and rest, can take advantage of a geometric-type analysis of the position and mobility of the system —an analysis, therefore, of incipient motion or virtual analysis, as we would now say— searching for the conditions that prevent such a motion: equilibrium is obtained from non-activated or blocked motion.

Quindi è che non deve sembrare che operi fuor di proposito, chi della fermezza degli archi e delle cupole si mette a trattare in una maniera quasi semplicemente teorica. Posto che un arco o una volta di qualsivoglia genere, costruita sia in guisa, che per la sua figura e forma, attese le leggi di gravità, le varie parti delle materie che la compongono aver debbano fra loro equilibrio, per parte degli sfregamenti e delle malte sarà tanto più allontanato il pericolo della caduta. (Mascheroni 1785, XXV).2

Furthermore, the analysis is finalised to the search of conditions which will keep the danger of collapse at bay: limit analysis, therefore, through the Principle of Virtual Works.

But what was the state of the art as Mascheroni knew it? He criticises the author of the Vite de’ più celebri Architetti, who, while agreeing with Frezer’s recommendation that workers should not put their trust in practice alone4, did not keep to this very theory that he recommended, but provided defective rules for the size of arches, vaults and cupolas, rules which were very remote from those that «Geometri di chiarissima fama di qua e di là dall’Alpi . . . hanno insegnato.» (Mascheroni 1785, XXVII).3

The names and writings of these Geometricians are quoted in the preface to Abbot Bossut’s Memoria published in the volume of the Paris Academy in 1774. Two years later Bossut deduced the general equation for the equilibrium of domes; subsequently Lorgna
published a new theory on curves found in vaults in the Petersburg Commentaries (1779); on the same subject he also resolved many elegant and useful problems in his «Saggi di Meccanica e Statica», printed in Verona in 1782. As Mascheroni says: «pieno di rispetto per questi illustri scrittori che mi hanno preceduto ed istruito, io rifletteva non ostante che molte cose ancora restavano da ricercare.» (Mascheroni 1785, XXVIII) 6

The open questions to which Mascheroni refers are aspects regarding the correct geometrical and mechanical description of the problems:

Nissuno di loro aveva insegnato la maniera di far passare la curva dell’equilibrio per i centri di gravità degli elementi di un arco solido, maniera per altro la più diretta e naturale per ottenere la total sicurezza dell’arco medesimo . . . Tutti avevano posto questa curva nella concavità interna dell’arco, che intradus da’ Francesi si suole con proprio vocabolo nominare. Essi non han fatto avvertire il pericolo della caduta dell’arco, che con un tale metodo di procedere molte volte s’incorre; molto meno determinarono quelle curve, che non lasciano mai l’arco esposto a simil pericolo . . . Tratto M. Couplet (Couplet 1731, 1732) la materia delle Volte piane, che piattabande si chiamano; ma non ne diede una teoria generale, e nello stesso caso particolare nel quale si impiega il cerchio per determinare le convergenze de’ tagli delle pietre, non fissò alcun limite alla lunghezza delle piattabande, dal che ne segue manifesto rischio di rovina . . . Tratto M. Couplet (Couplet 1731, 1732) la materia delle Volte piane, che piattabande si chiamano; ma non ne diede una teoria generale, e nello stesso caso particolare nel quale si impiega il cerchio per determinare le convergenze de’ tagli delle pietre, non fissò alcun limite alla lunghezza delle piattabande, dal che ne segue manifesto rischio di rovina . . . Ma quello che più richiedeva l’esame geometrico si erano gli archi e le volte composte. (Mascheroni 1785, XXVIII–XXXI) 7

We shall now proceed to dedicate our attention to the methodology proposed by Mascheroni for carrying out geometric examinations of arches. This process, based on axioms and hypotheses for the analysis of simple problems, evolves until it reaches the analysis and solution of Problems X and XI, the first formally correct example of limit analysis in the history of Mechanics. A cardinal point of this process is his geometric analysis of mobility, which reveals a complete knowledge of the characteristics of the act of motion of a rigid body.

THE GEOMETRIC PRINCIPLES OF MECHANICS AND STATICS IN MASCHERONI

On the equilibrium of Straight lines

Since Mathematicians consider curves as polygons with an infinite number of sides, before dealing with the curves which are required for the equilibrium of Vaults, Mascheroni first states some Problems on the Equilibrium of Straight lines. Such Problems, dealt with in part by Lorgna in his «Saggi di Meccanica e Statica» (Verona 1782), are taken up again not only to ensure complete treatment of the matter, but also to give new demonstrations of them. The problems are preceded by a few hypotheses which constitute the axiomatic premise of the analyses and theorems developed by Mascheroni.

The geometric-mechanical model: axioms and hypotheses

1) The motion under consideration is an infinitesimal movement of the first order: «. . . il moto è lungo una linea infinitesima, perché in tale ipotesi il moto segue con velocità uniforme per tutta la linea, il che è necessario per calcolare la quantità di moto. Inoltre, tale ipotesi è necessaria per ogni caso in cui il centro di gravità di un corpo muovendosi cambi continuamente direzione.» (Mascheroni 1785, 2). 8

2) Relative gravity is that which pushes a body along an inclined plane. The product of such gravity for the mass is to be called relative weight.

3) Equilibrium is born from the equality of forces, understood as a product of the weights for the relative velocities (Principle of Virtual Velocities). In fact, «. . . si considererà il rapporto che possono avere tra loro due forze contrarie facendo seguire per supposizione il moto di due centri di gravità lungo due linee infinitesime da una parte e dall’altra; uno dei due moti è effetto dell’altro.» (Mascheroni 1785, 2). 9

Theorem I or of the equilibrium of two weights connected by machine construction

Let two weights $p_a$ and $p_b$ be placed initially in $A$ and $B$ (Fig. 1). Consider the infinitesimal trajectory $Aa$ of $p_a$ upwards along an inclined plane and simultaneously, due to machine construction, the infinitesimal downwards trajectory $Bb$ of $p_b$. If the motion occurs in the same interval of unitary time, $Aa$ and $Bb$ represent the virtual velocities of $A$ and $B$. In addition,
The equilibrium of two weights connected by machine construction

let \( AH \) and \( Bh \) be the vertical projections of the trajectories \( Aa \) and \( Bb \). The equilibrium condition of the system requires that the products of the weights be equal for the corresponding vertical trajectories in the two opposite directions, and that is:

\[
p_A AH = p_B Bh
\]  

(1)

**Theorem II or Torricelli's Theorem**: If there is equilibrium, that is (1) holds, \( A \) and \( B \)'s centre of gravity does not descend.

**Theorem III or on the characteristics of a displacement of a rigid body**: «Se una linea retta fa un moto infinitamente piccolo qualunque, tutti i punti di essa linea fanno un viaggio eguale sulla direzione della medesima linea.» (Mascheroni 1785, 4-5).\(^{10}\)

Si consideri l’asta \( QM \) che, per effetto di uno spostamento infinitesimo, si porta in \( qm \). Abbassata dalla posizione variata di ogni punto la perpendicolare a \( QM \), e individuato lungo l’asta il punto \( H \) caratterizzato soltanto da uno spostamento parallelo alla configurazione iniziale \( QM \), si afferma che: \( Hg = He, Hq = Ha \) e quindi: \( gq = ea \), Figure 2. Puoi per caso esprimerti meglio in italiano?

Consider the rod \( QM \) which, as a result of an infinitesimal displacement, moves to \( qm \). Having lowered the perpendicular segment to \( QM \) from the varied position of each point, and having identified point \( H \) along the rod —this point being characterised only by a movement parallel to the initial configuration \( QM \) — one can affirm that: \( Hg = He, Hq = Ha \) and therefore: \( gq = ea \), Figure 2.

As has already been said, this implies that the movement of each point is an infinitesimal of the first order. In addition, the movement of each point can be considered as a vectorial sum of a constant displacement, along the direction that coincides with the initial rectilinear configuration (the translational component of the motion equal to that of the chosen pole) and of a displacement that is orthogonal to the direction, coinciding with the initial rectilinear configuration (the component of the motion due to rotation around the chosen pole).

Although this has not been demonstrated, the essential features of an infinitesimal displacement of a rigid body are fixed for the particular case in which a point \( H \) of the rod moves in the direction \( QM \).
parallel to the rod itself: this movement, which is an infinitesimal of the first order, can be considered the sum of a contribution from the translation, equal to the movement of the pole \( H \), and of a contribution from the rotation, orthogonal to the direction defined by \( H \) and the point under consideration.

**PROBLEM I OR OF THE EQUILIBRIUM OF WEIGHTS AT THE EXTREMITIES OF ARTICULATED RODS**

The system consists of due rigid rods \( CB \) and \( BA \) hinged in \( B \). A weight in \( B \) moves as a result of the rotation of the rod \( CB \) around \( C \), while a weight in \( A \) descends along a vertical trajectory. This is a first schematization of the collapse mechanism of a symmetric arch, with a formation of five hinges: one at the key extrados, two at the intrados of the rupture joints and two at the extrados of the springers. This simplified model ignores the weight of the rods and concentrates the weights at their extremities.

According to Theorem 1, equilibrium requires that the product of the weight \( p_b \) in \( B \), for its vertical upward trajectory \( p_b \), be equal to the product of the weight \( p_A \) in \( A \) for its vertical downward trajectory \( Aa \), Figure 3:

\[
p_b \cdot p_b = p_A \cdot Aa
\]

This demonstration, of a purely geometric kind, is based on the hypothesis that the displacement \( bB \) of \( B \) is orthogonal to \( CB \) (virtual displacement of a rigid body due to a rotation \( \delta \theta \) around the pole \( C \)), so that:

\[
p_b = CE \cdot \delta \theta = CE \cdot pB / BE
\]

The movement \( Aa \) of \( A \) is considered, on the other hand, to be the result of a vertical movement \( Vn \) due to the relative rotation of \( A \) around \( B \), from which the vertical movement \( p_b \) due to \( B \) must be subtracted, so that:

\[
Vn = BF \cdot \delta \theta = BF \cdot nA / AF = pB \cdot BF / AF
\]

and:

\[
Aa = Vn - p_b = (BF / AF - CE / BE) \cdot pB
\]

Thus, for equilibrium:

\[
p_b \cdot CE / BE = p_A \cdot (BF / AF - CE / BE)
\]

It is remarkable that the solution to this problem demonstrates a complete awareness of the fact that the absolute movement of \( A \) can be considered as an algebraic sum of the relative movement of \( A \) with regard to \( B \), plus the movement due to the dragging of \( B \).
**Problem II or of the Equilibrium of Heavy Hinged Rods**

The demonstration is substantially similar to that of Problem I, only this time the weights are concentrated in the respective centres of gravity Q and G of the two rods. Thus, Figure 4:

\[
p_g \frac{CK}{BE} = p_g \left(\frac{BT}{AF} - \frac{CE}{BE}\right) \tag{7}
\]

**Problem X or of the Equilibrium of Arches in the Presence of Infinite Friction**

On the basis of axioms, theorems I–II–III and the results supplied by the solutions to Problems I and II, Mascheroni was able to execute the limit analysis for Problems X and XI with coherence and rigour. Problem X is thus formulated as follows, Figure 5:

Supposto che nell’arco solido LHAONVBM posto sopra le due basi LCRM, NSTO il punto A che si trova alla sommità possa discendere perpendicolarmente aprendosi l’arco in V, e lateralmente in H e P, e ascendent i due punti B e X, restando sempre congiunte in un pezzo solo le parti HBVA, AVPX eguali, e parimenti le parti HLCRMB, XPOTS N, le quali debbano alzarsi col girare intorno a’ due centri C e T; seguendo il tutto come se in C, B, A, X, T vi fossero delle cerniere; trovare la ragione delle due forze. (Mascheroni 1785, 28).\textsuperscript{11}

Given the symmetry of the structure, the problem is analysed only for the semiarch. The solution, Figure 5, is obtained by using the same relationship obtained in Problem II. The limit situation of equilibrium therefore corresponds to:

\[
p_g \frac{CK}{BE} = p_g \left(\frac{BT}{AF} - \frac{CE}{BE}\right) \tag{8}
\]

**Problem XI or of the Equilibrium of Arches with Infinite Friction at the Springers and No Friction at the Rupture Joints**

This is the same collapse mechanism investigated by La Hire in 1712 and analysed again by Mascheroni according to the schema in Figure 6. The problem is formulated thus: «Supponendo che il pezzo di arco solido HBVXPAH (Figura 7) discenda perpendicolarmente, e parallelamente a sé stesso, e facendo cogli sdrucchiamenti de’ lati HB, PX alzare i due pezzi HCRB, PTSX giranti intorno i centri C e T; trovare la ragione delle forze contrarie». (Mascheroni 1785, 29).\textsuperscript{12}

Given the symmetry of the structure, the problem is once again analysed only for the semiarch. At collapse the keystone descends vertically, nonetheless maintaining contact at point B—the point of the rupture joint HB—with the lower voussoir rotating around C.

Since bB is orthogonal to CB according to Theorem III, from the solution of Problem II the vertical movements ba of point B and yQ of the centre of mass Q, due to the infinitesimal rotation \(\delta \theta\) of the springing voussoir around C, are respectively, Figure 7:

\[
ba = CE \delta \theta_1 = CE \frac{Ba}{BE} \tag{9}
\]

\[
y_Q = CK \delta \theta_1 = CK \frac{Ba}{BE} \tag{10}
\]

The vertical movement of all the keystone points, and therefore also of the centre of mass G, is considered to be an algebraic sum of a relative vertical downwards movement ea of G with regard to B, plus the upwards dragging movement ab of B. That is:

\[
y_g = eb = ea - ab = Ba FN / BF - Ba \frac{CE}{BE} \tag{11}
\]

---

**Figure 5**

**Figure 6**
The limit condition for equilibrium therefore corresponds to:

$$p_g \frac{CK}{BE} = p_G (\frac{FN}{BF} - \frac{CE}{BE}) \quad (12)$$

It is interesting to compare the solution Mascheroni proposes with that previously proposed by La Hire. This too was based on Geometry: «On a cru qu’il fallait chercher dans la Géometrie une regle sur laquelle on a pu s’assurer, pour determiner la force dont on doit faire les pieds-droits» (La Hire 1712, 69).

With regard to Figure 7, to determine the limit condition for equilibrium La Hire takes the problem back to that of an equivalent simple machine, that is, of an angled lever with its fulcrum in $H$. The arms of this lever are $HT$ and $HL$, at the extremities of which lie the weight of the abutment and the thrust $D$, respectively; the thrust $D$ is orthogonal to $HL$ and its role is equivalent to that of the weight $p_G$ of the key wedge, which descends vertically, sliding without friction on the rupture joint $ML$. The crucial point is how to determine the thrust $D$. At this point La Hire states: «on sait par la Mecanique que» (La Hire 1712, 72).

$$LG : CG = p_G : D \quad (13)$$

The proportion proposed by La Hire to calculate the thrust $D$ appears to belong to consolidated knowledge—but what can the thought processes be that lead to consideration of the segments $LG$ and $CG$?

It is only after having read Mascheroni that this proportion, so apparently obscure, can be justified. Above all, the axioms that Mascheroni establishes as fundamental to his theory consider it an acquired truth that, in a system made up of two weights joined by machine construction, equilibrium is guaranteed if the product of the weights—which they are absolute or relative, that is in movement along vertical trajectories or inclined planes—for the corresponding virtual velocities is balanced. In fact, La Hire’s problem seems to proceed from that of the two weights joined by machine construction: the first moves on an inclined plane corresponding to an upward movement $LG$, and has a relative weight $p_G$; the second weight $p_G$ is that of the key voussoir which moves vertically, sliding along the rupture joint $ML$. The machine construction that links $D$’s and $p_G$’s movements is the condition for the maintenance of contact in $L$ between the wedge and lower voussoir.

It is therefore surprising to discover that La Hire was well aware that while the weight $D$ moves upwards along $LG$, the wedge maintaining contact in $L$ makes a vertical movement that is the algebraic sum of the vertical downwards movement $EC$ —the vertical relative movement of the voussoir with regard to $L$—and of the vertical upwards movement $GE$—dragging movement of $L$.

It would seem then that knowledge of geometry and in particular that of kinematic characteristics in the so-called mechanical questions was far more widespread and well-established than one would assume from the analysis of historical texts. Not only, but as asserted by Mascheroni in his preface to the Nuove ricerche sull’equilibrio delle volte (Mascheroni 1785), the role of Geometry understood as geometry of movement, was the real area of research in Mechanics, with the Principle of Virtual Velocities having been assumed as the unique principle of reference. It is with Mascheroni that this kind of geometric knowledge first acquires its systematic organisation of an axiomatic kind.

Figure 7
La Hire’s collapse mechanism
NOTES

1. «... are arts oriented to functions: the result of the complication of multiple circumstances that alter the status of the question is that only speculation succeeds in reaching decisions that are extremely removed from practical experience, and so it is easy to see that theory must predominate in Statics; contrariwise practice should be taken into consideration as far as Mechanics is concerned. This latter while engendering movement finds hindrances in the resistances of the means, in the roughness of surfaces, in the tenacity and various attractions of the materials. These kind of circumstances are not easily identified by means of principles, and even less can they be subjected to the precision of calculus. Nevertheless, as they must perforce be taken even less can they be subjected to the precision of attractions of the materials. These kind of circumstances, even distinguished Geometricians have ever less did they calculate the curves that would never leave the arch exposed to such danger... M. Couplet dealt with the matter of plane Vaults, which are called platbands; but he did not provide a general theory about them, and in the particular case in which a circle is used to determine the convergence of the cuts of stones, he did not fix any limit to the length of the platbands, which leads to a clear risk of collapse... Above all it was the arches and composite vaults that required geometric examination.»

2. «Statics, on the other hand, in its attempt to introduce equilibrium and quiet into various groups of bodies, is aided in its intent by the very things that contrast with Mechanics; so that whenever Mechanics, with the help of Geometry, has found its site and the distribution of those materials which should stay still, in the above-mentioned circumstances it cannot but be made even more secure.»

3. «It should not therefore appear that someone is working out of context when he treats the stability of arches and cupolas in an almost simply theoretical way. Given that an arch or cupola of any kind is built in such a way that by its shape and form, in accordance with the laws of gravity, the various parts of the materials that compose it must be in equilibrium with each other; and with the addition of friction and mortar the danger of collapse will be even further averted.»

4. «Even the longest practical experience is not sufficient for the correct building of Vaults. In this matter an old experienced man is an old ignorant man who can easily be mistaken even in small matters, the cases vary. In this particular matter there can be an infinity of cases in which the reasonings that a practical man may make after carrying out a work prove to be fallacious. Forty-six years of practice without theory could not teach the architect who, in a French border town in 1732, had to build a powder magazine —as he did not make the abutments of the correct thickness, the building collapsed before the scaffolding was removed.» (Autore delle Vite de' più celebri Architetti, 1768, part II, chap.1, Mascheroni 1785, XXVI)

5. «... extremely famous Geometricians from both south and north of the Alps... have taught us»

6. «... while fully respecting these illustrious writers who preceded and instructed me, I thought nonetheless that many things still remained to be discovered.»

7. «None of them had been taught how to make the curve of the equilibrium pass through the centres of gravity of the elements of a solid arch —a method which is also the most direct and natural for obtaining total security of the arch itself... All of them had placed this curve in the internal concavity of the arch, which the French call intrados. They had not warned of the danger of collapse of the arch, which with this procedure often occurs; even less did they calculate the curves that would never leave the arch exposed to such danger... M. Couplet dealt with the matter of plane Vaults, which are called platbands; but he did not provide a general theory about them, and in the particular case in which a circle is used to determine the convergence of the cuts of stones, he did not fix any limit to the length of the platbands, which leads to a clear risk of collapse... Above all it was the arches and composite vaults that required geometric examination.»

8. «... the motion is along an infinitesimalline, because according to this hypothesis motion occurs at a uniform velocity along all the line —this is necessary if one is to calculate the quantity of motion. In addition, this hypothesis is necessary for all cases in which the centre of gravity of a moving body constantly changes direction.»

9. «... we will consider the possible relationship between two opposite forces, under the supposition of making the motion of two centres of gravity follow along two infinitesimal lines on one side and the other; one of the two motions is the effect of the other.»

10. «If a straight line makes any kind of infinitely small movement, all the points on that line travel equally in the direction of the line itself.»

11. «Supposing that in the solid arch LHAONVBM placed above the two bases LCRM, NSTO the point A at the top can descend perpendicularly opening the arch in V, and laterally in H and P, with the two points B and X ascending (rising), the equal parts HBVA, AVPX remaining joined in one piece only, as also the parts HLCRMB, XPOTSN, which must rise as they turn around the two centres C and T, following everything as if there were hinges in C, B, A, X, T, find the ratio between the two forces.»

12. «Supposing that the piece of solid arch HBVXPAH (Figure 7) descends perpendicularly, and parallel to itself, and by the slipping of the sides HB and PX causing the two pieces HCRB, PTSX rotating around the centres C and T to rise, find the ratio between the opposing forces.»
The role of geometry in the theories on vaulted structures by Lorenzo Mascheroni

13. «It was thought that one had to search for a rule in Geometry on which one could rely, to determine the strength with which the abutments should be made.»
14. «It is known from Mechanics that . . . .»

REFERENCE LIST
