A spherical vault, whether a hemisphere or a sail vault, is formed of voussoirs. The voussoirs are typically arranged along circular horizontal rows, separated from each other by conical surfaces that are more closed as we advance toward the pole and whose vertex is always the center of the sphere. Within the same row the voussoirs are all equal or similar and they are separated by joints following vertical planes which converge on the central axis. In this way each piece has a spherical inner surface, two top and bottom bed joints that are cones, and two joints that are vertical planes. In the unusual case that the piece appears on the extrados (exterior) of the vault, then the exterior surface could be spherical as well (figure 1).

We find many exceptions to this geometrical division, though most involve slight variations from arranging the parallels and meridians in a different spatial position (Fig. 2). In effect, the sail vaults — those that are sectioned by four perimeter arches in order to adapt to a square foundation — on occasion present a geometry that in the 16th century was named «por hiladas cuadradas» (for its square courses), and that, as its name indicates, arranges rows in a square horizontal projection, each time smaller until closing the surface. Each of the sides of these square courses is a vertical row equal in all ways to those described above. So we have here a division also according to parallels and meridians, although with the axis placed horizontally.1

The usual procedure in the 16th century for the carving of the voussoirs of a spherical vault is as follows. Over a block of stone from the quarry, one would first carve the inner concave surface of the vault (intrados), the one which will remain visible, but not starting from its definitive edges, but rather working a portion of spherical surface, to afterwards mark on it the real contour. This operation, the definition of the contour of that face, would take place applying over the face a template, as is typical in the work of stonecutting. However, as is known, a spherical surface is not developable, so the application of a flat template over the sphere is only approximate. In any case, through this one would proceed to the carving of perimeter, bed and vertical joint surfaces.
As we have said, this involves different surfaces, two cones and two planes, but all can be considered as surfaces generated by the movement of a radius of the sphere, following parallels in the case of the conical beds and meridians in the case of the planes. Therefore these four surfaces can be carved with ease employing an instrument called a bevel. The bevel is a type of square that has a curved piece that adapts to the concavity of the inner surface and another straight branch that follows the direction of the bed joint towards the center. Its use is common in the carving of the voussoirs of the arches, as it allows proof of the orthogonality of the inner surface and the bed joint and the correct execution of both. Its application to the spherical voussoirs is a brilliant idea, as it determines the shape of more complicated pieces than those of an arch.

Finally, if it were necessary to carve the exterior, one would mark its contour tracing a line parallel to that of the intrados and confirming the carving with a concave curved ruler or a concave bevel.

This procedure is of the type that we call direct, as opposed to those that pass through a previous squared-up stone beforehand. In the 18th century variations on the use of the template for the spherical voussoirs were proposed and also the procedure of the squared-up stone was proposed, but there is no doubt that in Spain in the 16th century the usual procedure was that which I just explained. That is what is shown in the manuscript of Alonso de Vandevilra, and with more detail in that of Alonso de Guardia, at the end of the century or beginning of the next, and the book of Francois Derand or the manuscript of the Catalan Joseph Gelabert, in mid-17th century.2

We now return to the template of the intrados. Made up of brass or cardboard, it should allow a certain curvature to adapt to, although imperfectly, the spherical surface where it is applied. It is in reality a small piece of an approximated sphere. A sphere can be developed approximately if we suppose that it is divided in segments according to meridians (Fig. 3), and we liken each one of these segments to a portion of the cylinder. It is a procedure employed in cartography. We can also divide the sphere in horizontal sections according to the parallels, and substitute, in each sector, the spherical surface for the trunk of cone. These cones, more or less open, have their vertex always on the axis of the vault. This is the

Figure 2
Various arrangements of the joint according to parallels and meridians.

Figure 3
Developments of the sphere according to cylinders and cones.
approximation chosen by the stonecutters (the approximation of the segments also will be employed in certain occasions, especially for the design of the decoration.) Each one of those cone sections are easily developed tracing the corresponding arches with the center in its vertex (Fig. 4). The template of each vousoir is no more than one part, more or less large, of this development.

Figure 4
Layout for a dome in the treaty of Vandelvira

The prepared template would be applied over the spherical inner surface of the vault to mark its contour. If we were very strict with the geometric concepts, we should take care that the template touches the surface only in the edges corresponding to the parallels, remaining separate along of the other two sides (fig. 5). But the practice shows that, even in vaults of small span and great curve, this precaution is excessive, and the stone-cutter commits a little error if he applies the template flattening it without care so that it remains in contact with the stone.3

Thus it happens that, finally, the use of the template and the bevel very easily solves the work of carving the vousoirs for spherical vaults. The stone-cutter would require a different template for each row, but the templates, and the pieces, of a single row are all the same. Even if the vault is not very small, the vousoirs of each row can be of different longitudes, according to the dimensions of the blocks that the quarry provides, but its templates always have the same form, more or less extended (or applied successively).4

There are another type of hemispherical vault, that presents just one helicoidal row, named by Philibert Delorme «en forme d’une coquille de Limaçon» (De l’Orme 1567), and by Alonso de Vandelvira «en vuelta de capazo» (Barbe-Coqueline 1977). However these two authors do not develop the problem in the same way.

The concept is, in principle, different from those explained, because it starts from another possible development of the sphere. It involves something easily conceivable in so much as it is similar to the peeling of a fruit. From the geometrical point of view it is something more complicated. As before when we signaled parallels or meridians and between them we substituted the sphere for cones and cylinders, now we should trace a helicoidal line over the surface of the sphere,—something that can be done a few
different ways—, and to support between each two turns of the helix a ruled developable surface—for which there is only one solution, but is not as simple nor evident as when it involves cones and cylinders.

The drawing that we find in the vault *en limacon* or like a snail, of Delorme (figure 6), reappears almost exactly in the manuscript of Jean Chereau (1567–1574) (figure 7). Chereau copies many solutions from Delorme; in this case it is done in a literal way, evidently without understanding the problem, as it duplicates the same mistakes.

In effect, the two drawings trace a spiral in plan, and that spiral is projected vertically in the spherical surface. The spiral of the base is drawn in such a way that the progression of the line in each revolution is constant. Consequently, the height of each row is continually variable. As happens in other layouts of hemispherical vaults, in the layout of this vault we have the benefit of the same circumference for the base and the section, in such a way that in the upper

![Figure 6](image6.png)

Figure 6
The vault «en forme d’ une coquille de Limacon» by De l’Orme

![Figure 7](image7.png)

Figure 7
The vault *en limacon* in the manuscript of Jean Chereau
The single coursed ashlar vault

left quadrant we can see the semisection, that projects the partitions of the base vertically upwards, and shows that the height of the row increases from the pole towards the equator. Precisely from here is where the most important error of the layout originates: the heights of the rows resulting in the upper part, numbered 6, 5, 4, 3, are different but moderately so, whereas the height that would result in the base of the vault is notably greater than the rest, so that the author (both authors) have decided to divide this first height (1 and 2) in order that it becomes similar to the rest. Evidently this is not a well-defined solution, because, although we decide to begin with that divided row, at some point we will have to return to just one piece, etc.

It is certain that the text of Delorme alludes to the possibility of employing the same layout to design of a conical or spherical vault — although the text speaks of a «pyramidal» form, it is to be assumed that it refers to a cone, as it involves, it says, covering a tour ronde or a spiral staircase. In the case of the cone, the spiral of the base would have projected onto something very approximate to an ordinary helix of constant step. But the drawing clearly develops the spherical option.

So then, the drawings of Delorme and Chereau contain operations that any craftsman who really faced the problem would have rejected: to think about the spiral directly like a tracing in plan then transferred to the space, to accept a row of constantly variable thickness, and to solve the problems that these decisions generate with unskilled and undefined clumsiness.

We examine now the layout of Alonso de Vandelvira (fig. 8). This author draws the spiral of the base from the point of the spatial helix that he wants to obtain. The process consists of first determining the height of the row by dividing the section as if it involved a conventional hemispherical vault in order to afterwards make those points of division rotate around the axis at the same time that they uniformly advance towards the pole by their meridian. Vandelvira divides the circle of the base in equal parts (16 of them) and divides the height of the row in the same number of parts, projecting them horizontally, in order to trace the spiral line in plan. Each unit of revolution corresponds to one part of nearing the center. The layout is, naturally, a flat drawing, but it corresponds with a spatial conception of the line.

As a consequence, in Vandelvira’s layout the height or distance between the lines of the upper and lower bed joints is constant. This helicoidal solution of the spherical vault is characterized by its unnecessary complication, in that it is unavoidable that all of the pieces will be different; but, within this general inconvenience, Vandelvira’s layout departs from reasonable assumptions and attempts a certain uniformity, while that of Delorme arrives at an absurd solution.

On the other hand, neither Delorme nor Chereau say anything concerning the templates for the execution of the pieces. Delorme draws them, and it seems that he obtains them starting from a theoretical template of conventional model, but, as is common for Delorme, he doesn’t explain anything. Although
the layout that Vandelvira proposes to obtain the templates could be criticized from a strictly geometric point of view, at least the author undertakes and clearly explains this theme.

In effect, we have confirmed that the template for the carving of a conventional spherical vault results from the development of a section of cone approximating the spherical surface. In this case there is no cone as a substitute for a portion of the sphere. It is unlikely that Vandelvira conceived of the ruled developed surface mentioned before as a possible approximation to the sphere. So that, in this case, as in others of his treaty, he would obtain the more or less quadrangular template in a very approximate way, looking for the distances in real magnitude between the vertices and joining these vertices with lines reasonably near to the straight or curved lines that we intuitively should obtain.

The intrados have four corners, four vertices in its template, that will not be over the same plane. After estimating the distances of the four sides, we will have to choose one of the two diagonals, as each one will result in a different quadrangle. Consequently, what Vandelvira does is not a development, but nor can it be said that it is the real form of the relative positions of the vertices. The obtained template could be of value if it is folded by one of its diagonals, keeping the two parts level, about which Vandelvira says nothing. Therefore, the triangulation that Vandelvira uses in order to draw the relative positions of those four points implies a geometrical approximation. On the other hand, this template will be similar to the conventional in that the lines of the lateral joints will appear straight, but the superior and inferior sides, corresponding to the bed joints, are now developed from the helicoidal line and not from circular arcs. Vandelvira draws them, however, as circular arcs, and it is curious to note that, in doing this, he does not consider something similar to the cone that passes through the superior and inferior lines, rather he invents a hypothetical tangent cone for each one of two parts of the helicoidal line. As a consequence, the superior edge of a template coincides with the inferior edge of the template of the piece that goes above it, which theoretically could not occur, as Vandelvira occupied himself with explaining in reference to the hemispherical vault.

It should be recognized that Vandelvira takes many licenses and departs much from what we now would understand as a strictly correct solution, but also that the posed problem was enormously difficult given the knowledge of that time, and, in this situation, Vandelvira makes a correct use of some resources, finding real magnitudes, imagining hypothetical cones, etc.

In fact it is very notable that, when the conceptual tools capable of solving this problem did become available in the 19th century, in contrary to what we would expect, no one attempted the problem at all. The helicoidal joints of the dome are not even mentioned. We might assume that the stereotomy of the 19th century rejected undertaking an unnecessarily complicated problem; however it is clear that they did engage in studying other rather absurd and sophisticated proposals. The reality is that the correct execution, which demands the determination of the ruled surface and its development, would be complex and not very elegant from the geometric point of view.

It is worth noting that this type of vault appears in the copy of the treaty of Vandelvira conserved in the Escuela de Arquitectura de Madrid, but not in the copy kept at the Biblioteca Nacional (by Felipe Lazaro de Goiti). Vandelvira presents this vault as a variant of the vuelta redonda or conventional hemispherical vault, and in the copy at the Escuela de Arquitectura it appears after the collection of ribbed hemispheres. The copy of Felipe Lazaro de Goiti omitted all of the ribbed vaults, as much the gothic ones as the rib and panel vaults (enrejada), probably thinking that they had to do with an old tradition. If this type of vault (vuelta de capazo) was situated in the original as it is in the copy at the Escuela de Arquitectura, that is to say, a continuation of the rib and panel vault hemispheres, it is not strange that it would be left out together with this group.

Perouse de Montclos (1981) explains that this particular method and other similar games are no more than unrealized fantasies of Delorme and Chereau. It is certain that we do not know of any French examples, and, in any case, it is evident that the drawing of Delorme/Chereau can not have any relation with a real example that was known very directly by them, because as I have argued, it presents important practical problems.

Similar layouts of spirals in a circle are found in the manuscript of Hernan Ruiz el Joven; it is not clear, however, if this is a layout for a vault of this type,
since it involves Archimedes’ spiral (in some cases the integration of a few), that open or separate progressively as they depart from the center, which would notably exaggerate the defect found in Delorme. It would be possible, however, to have a relation with the ionic volute; in fact one of these spirals is applied to this function. In any case, the usual layout of the ionic volute is a spiral ready-made with circumferential arcs, of diverse centers, starting from an initial small circumference.

I have commented that the way to trace the spiral of the base in Delorme/Chereau allows the step to be constant. This is done with a layout very similar to that of the ionic volute, employing circumferential arcs. In this case it started from the circle of the central keystone, employing the two points of the circle that coincided with the horizontal axis. The spiral begins then with a first semicircular arc from the center of the left, continues with another semicircular arc from the center right, etc. It is easy to verify that in this way the distance between turns is constant and equal to double the diameter of the keystone. We can also find a spiral of two centers in Villard de Honnecourt’s notebook, which has been put in connection with the layouts of pointed arches (Bechmann 1991, 225–230).

The type of the vault de capazo (or en limaçon) appears later in the book of Milliet-Descales (1674), figure 9, and in that of T. V. Tosca (1707), figure 10. It is known that Tosca also copies Milliet-Deschales in almost all of his exposition on the geometry of stonecutting. In this case the original French layout is as problematic as that of Delorme, and Tosca’s copy is even worse.

In effect, Milliet-Deschales lays out a spiral even more similar to that of the ionic volute, as it employs four centers, the four axial points of the initial circle, but in doing this obtains, as in the case of the volute, a progressively wider step as it departs from the center. In this way the problem of the row’s height is made worse, and the author no longer occupies himself with drawing the section, where again the first row would appear disproportionately taller than the rest.

Tosca copied the drawing, which is identical in a multitude of details, but takes two centers, and these points are, unlike those of Delorme, one on the cercle (that is marked 1) and the other in his center. In this way there are not four circumferential arcs in each
turn, rather only two, and one of them is traced from the center of the circumference, producing concentric semicircles with it. So then, the spiral form is achieved in Tosca in the strangest of ways, since the half of the base is exactly like a vault of conventional round rows, while the other half is charged with constructing the helices. The text just refer to the layouts of conventional spirals, as if the subject were irrelevant.

These two authors draw the template of the vault faces by a somewhat different process to that of Vandelvira, although also approximate, departing from an ideal straight template. The Spanish manuscript of Portor y Castro,6 copying Tosca, offers the same drawing (Fig. 11). I do not know of other layouts for this pattern.

Figure 11
Hemisphere that closes in «a manera de rosca o de linea espiral» (in the way of rosca or of a spiral line) by Juan de Portor y Castro

So then, this suggests a stereotomy method conducted by the French theorists, in all cases probably from the point of their diffusion by Delorme, copying it (Chereau) or trying to improve it (Milliet-Deschaes). A certain branch derives from Milliet-Deschaes by the copy of Tosca and of Portor. It is difficult that none of these authors really put into practice their execution, given the removal from reality that their processes show. The layout of Vandelvira, without any relation to the former ones, is on the contrary, that of someone who perfectly dominates the real problems of this pattern.

In a space next to the the vestry (antesacristia) of the Murcia cathedral we find an executed example of this type of vault (de capazo), figure 12, that shows a row of constant height, like in the case of the layout of Vandelvira. Its surface is «wrinkled», as it was intended to receive a subsequent finishing, except in a detached molding along the helicoidal line of the bed joints. Perhaps this concealment of the radial lines was meant to cover the small defects that a layout as problematic and necessarily approximate would need to endure.

Figure 12
Vault de capazo in the Murcia cathedral

Despite the geometrical analysis that I have made, we have to recognize that, as much in the conventional round row semi sphere as in this helicoidal variant, the real and practical demand of precision in the form of the pieces is not required. The French solutions and their derivatives committed such important errors that it is difficult to imagine a
practical solution along such models. But that of 
Vandelvira, despite its multiple approximations and 
licenses, offers a buildable pattern. This is especially 
true if we keep in mind that accepting approximate 
solutions was a habitual custom in the stonecutting of 
the 16th and 17th centuries.

Greater demand for precision is required for another 
of the examples of this type of vault found in Spain, 
the one situated in the spiral stairway, named caracol 
de Mallorca, that leads to the right tower of the 
Palace of the Guzmanes de Leon, figures 13 and 14. 
The two towers of this principal façade are recent 
reconstructions of a somewhat austere aspect, but the 
stairway, crowned by a vault with a decorated 
hanging keystone, is evidently original, and therefore 
dated to the second half of the 16th century and 
subsequently to the Murcia. As its dimension is much 
less than that in Murcia, the shaping of the voussoirs 

![Figure 13](image1.png) 
Spiral staircase in the Palace of the Guzmanes de Leon

Figure 14 
Dome that covers the staircase

demands somewhat more precision. In this case, the 
helicoidal line of caracol de Mallorca continues 
ideally in the covering that finishes it off. It is notable 
that the text of Delorme makes mention of the 
possibility of covering a round tower or a spiral 
staircase with these vaults. And that the layout of 
Chereau presents this vault on the same page as the 
vis de Saint Gilles, spiral staircase covered with a 
helicoidal barrel vault.

Finally we know of another even later example. It 
Involves the church of San Juan de los Caballeros in 
Jerez de la Frontera (Pinto 1998), figure 15. The vault 
is situated in the portico and covers a rectangular 
space of 1:2 proportions, so that the helix is found 
whole only in the very small central part. But its 
greatest peculiarity is that it doesn’t involve just one, 
but rather two simultaneous helicoidal rows, that 
arrive at the center, avoiding therefore a polar piece.
On the other side, in this same façade we find a most curious example of *vis de Saint Gilles*. So the relation between the spiral staircase and the vault *de capazo* continues.

We don't know the origin of these patterns. Although the proposal of Vandelvira may be much better and more real than that of Delorme, nothing impedes that it be this drawing first that, widely published, would provoke interest in Spain. It doesn't seem that the Florentine tradition of drawing helices in the brick domes would be relevant; it involves loxodromical curves generated by a herringbone pattern, and they are different from the vault *de capazo* as much constructively as formally. We should consider, however, two other possible influences. On one hand the Byzantine tradition of execution of spherical vaults with ceramic pieces prepared along one or various helicoidal rows. This is, for example the dome of San Vital de Ravenna. And furthermore we know of the existence of an example in stone much earlier, in Anatolia.

In effect, the Sultan Han near Aksaray (1229), figure 16, was built as a rest area for the caravans, with commercial and religious function. As usual for this type of construction, it has a bedroom space of great dimensions, similar to the Christian temples, and formed in this case by five naves separated by columns and covered by barrel vaults. In the center of the principal nave was constructed a vault of this type, over squinch arches covered with a pyramid also of stone. Its architect was originally from Damascus; it is unavoidable to record here the opinion of Viollet-le-Duc, who was convinced that all Western stereotomy had its origin in the admiration of traveling Christians towards the patterns of the Syrian construction. There are at least two other small domes or calotas, of reduced dimensions, with this type of pattern, covering the central part of domes in the hospital annex to the mosque of Divrigi, that date also to the 13th century.7

Finally we have to remember that this type of pattern is a solution enormously more complex than the conventional solution for a hemispherical vault. In effect, the solution by round rows demands only one template per row, and in each row the pieces are equal. Common sense leads Vandelvira to adopt a uniform height for the helicoidal row, but even so the pieces and all of its templates are different and require an individualized layout. It is to be supposed that in practice the constructed vaults were made with an added license to those other geometrical licenses already explained: since the design of all of the templates would be laborious, complicated and susceptible to confusion, one would employ the same template to a whole series, maybe to all of the ashlars in one turn of the helix.
In any case, the idea of the helicoidal row, which is reasonable in brick or ceramic pieces, becomes unnecessarily complicated in stone. That isn't the only case. Often more complicated and difficult patterns than the conventional ones have been proposed. But this, which happened especially in the 19th century, frequently was done with the intention of showing an elegant idea from the geometric point of view. Or rather, as occurs with the curious variants of waterspouts between the 16th and the 18th centuries, in order to develop a showpiece of a mechanical more than formal type. In this scope the vault de capazo is a very early and atypical example of this unnecessary complication. Maybe the intention of introducing the helix archetype in the stone work existed in some examples, but very probably the mentioned cases are displays directed to the stone-cutting guild itself.

NOTES

1. If one were to look for a difference, it could be found in the corners where the pieces, in the shape of L, correspond to two rows. Although frequently, including at El Escorial, this form is avoided and the pieces of the corners would be resolved in a similar manner to the rest.
2. I have justified these statements in Rabasa (2000).
3. I have confirmed that this is true in the construction of a small dome in collaboration with the Centro de los Oficios de León. It had only a one meter span, but for that very reason the pieces have great curvature.
4. If all of the voussoirs of a row have the same length, the template is made for this size. But if the dome is large, the distribution of joints is not established in advance, and each piece has a different length, adapted to the size of the blocks from the quarry. In this case the template is easily slid over the surface of the stone to mark the perimeter of the face of intrados when the length varies.
5. It is the manuscript Ms.12,719 of the Biblioteca Nacional de Madrid and R.10 of the Biblioteca de la Escuela de Arquitectura de Madrid, that is a reproduced facsimile by Barbe-Coquelin (1977).
6. Manuscript from 1708, conserved in the Biblioteca Nacional de Madrid with the call number Ms. 9114.
7. I am grateful to Profesor Aysil Yavuz for calling this to my attention. Drawings of these vaults can be seen in Yavuz (1993, 165–192).

REFERENCE LIST