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Before 1695: The statics of arches between France and Italy

Antonio Becchi

A POST-DATED HISTORIOGRAPHY

The specialist literature indicates *Proposition 125* of Philippe de La Hire's *Traité de mécanique* (La Hire 1695) as the first contribution to the subject of the statics of arches, looked upon as a problem of mechanics applied to construction. It is well known that La Hire swept away the empirical rules known until then and which enabled the stability of an arch be determined on the basis of the width of the opening and, occasionally, of the height of its supporting piers. *Proposition 125* was followed by the formulations in terms of analogy between the equilibrium of an arch and that of a catenary (already guessed at by La Hire and investigated in further depth by Gregory, Bernoulli and Stirling) and by La Hire's own formulation dated 27th February 1712 (La Hire 1712), with the collapse analysis of an arch. Scholars are in agreement as to this linear historical genesis,[1] which focuses attention on the *Traité* of 1695, the turning point that overcame the intuitions of Leonardo[2] and the «building site rules» recalled by Gil de Hontañon, Martínez de Aranda, Derand and Blondel.

We wish to show here that the historical reconstruction referred to above ignores at least two very important texts: the comment to *Quaestio XVI*, contained in Bernardino Baldi's *In mechanica Aristotelis problemata exercitationes* (published posthumously in 1621) and the dissertation *Remarques sur l'épaisseur qu'on doit donner aux pieds droits des voutes et aux murs des dômes ou voutes de four*, read and delivered by La Hire to the *Académie d'Architecture de Paris* on 27th October 1692. These texts force us to review the assessments taken for granted up to now and to re-examine more closely the relationship between mechanics and architecture in the 16th and 17th century.[3]

BERNARDINO BALDI'S *EXERCITATIONES*

Bernardino Baldi (1553–1617) tackled the problem of arches in the course of an extensive and original comment to Aristotle's *Mechanical Problems* (Baldi 1621). Baldi's *Exercitationes* are not mentioned by La Hire, nor even by other authors who deal with the topic of arches in the 18th century, such as Danyzy, Frézier or Coulomb. Neglected by those concerned with *Construction History* and read inattentively by the historians of mechanics, it was probably the first printed text in which the subject of mechanics applied to architecture was tackled systematically and in which a clear configuration of the collapse mechanism of arches was suggested. This detail alone would be sufficient to make the *Exercitationes* very interesting, however there are at least two more aspects that should prompt a careful reading of the text: the originality of Baldi's approach to the Aristotelian *Problems*, which were discussed at length in the 16th and 17th centuries, and the singular way in which the treatise was developed with

reference to *resistentia solidorum*, which with great intuition linked *Quaestio XVI* to the subject of the solidity and of the thrust of arches.

> The Aristotelian *Quaestio* is well known. It is quoted here in the Latin translation (Baldi 1621, 95): «Dubitatur, quare, quò longiora sunt ligna, tanto imbecilliora fiant, et si tolluntur, inflectuntur magis: tametsi quod breve est ceu bicubìtum fuerit, tenue, quod verò cubitorum centum crassum?».[4]

It is helpful to remember that the *Quaestio* itself constituted the problematical backdrop of the Second Day of Galileo's *Discorsi e dimostrazioni matematiche* (1638), a text considered to be the basis for the whole of the great chapter of mechanics devoted to *resistentia solidorum*. It is referred to explicitly by Simplicio on the subject of solids having the same resistance, and the echo of that query can be heard throughout the Second Day, during which, however, the theory of arches was not mentioned. For Baldi, on the contrary, the Aristotelian posit was the natural starting point of an excursus which knows no equals in the albeit substantial bibliography referred to the *Mechanical Problems* and which tackled in twenty pages important problems of the statics of constructions. To find something similar, many years would have to pass and all the contributions scattered in a large number of dissertations drafted in the late 17th and early 19th centuries would have to be pieced together.

After tackling directly the problem of thin rods, which is the subject matter of the Aristotelian *Quaestio*, Baldi extended his analysis to topics of other kinds: a column bearing a weight and, generally speaking, the distribution of weight on a supporting surface; the collapse mechanism of the beam of a floor; the solidity of roof trusses and of lintels. The latter brings up spontaneously the problem of arches, to which approximately half the comment referred to *Quaestio XVI* was devoted, showing that he had a specific interest in this subject. For reasons of space it is not possible to tackle here the entire treatment provided by Baldi. This paper will therefore be limited to an analysis of the collapse mechanism (Baldi 1621, 112–114).

With reference to Figure 1, Baldi argued that a semi-circular arch ABC will tend to break following a divarication of the supporting piers and, consequently, of the two semi-arches AB and BC. Once this displacement has occurred it is possible to identify two stable parts, AQ and CR, in the semi-arches. These two stable parts correspond to a tripartition of the original arch featuring identical angles and which, together, form therefore two thirds of the complete arch. The stability of these elements is taken for granted by Baldi in an earlier passage (Baldi 1621, 109), in which the centres of gravity of the elements AQ and CR (Figure 2) are identified in D and H, on the perpendiculars to the supporting surface passing through A and C respectively.

The location of the *centrum gravitatis* on these perpendiculars subsequently enables Baldi to neglect the contribution towards thrust provided by the elements AQ and CR, so that he can concentrate instead on the central parts.

QB and BR tend to fall rotating around the intrados points Q and R (Figure 1). This rotation can be prevented in part if the distance QR does not exceed

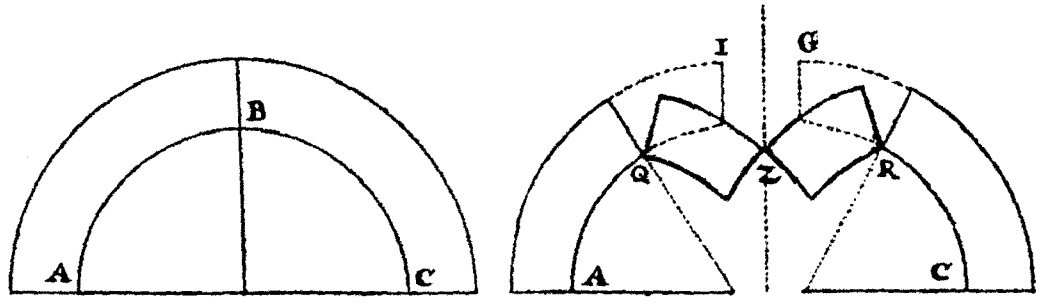

Figure 1
From (Baldi 1621), redrafted

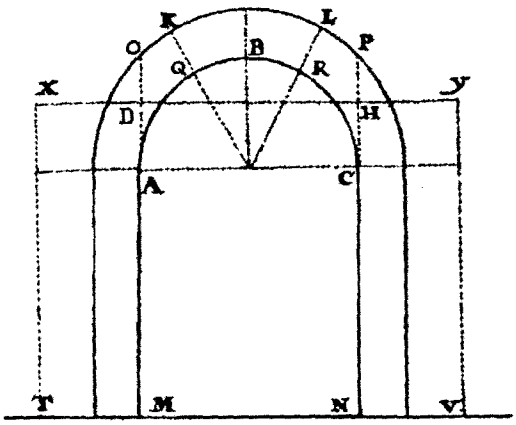

Figure 2
From (Baldi 1621), redrafted

the sum of the segments QI and RG, and it reaches a limit position when the two vertices I and G meet at point Z. According to Baldi, this collapse mechanism shows why thicker arches are more solid: indeed, in case of a thicker arch, full rotation of the central elements will only be possible in the event of a divarication of the imposts greater than that required for the previously discussed arch.

It is easy to see that the mechanical aspect of this «demonstration» can hardly be agreed with, but it does highlight three important aspects of the issue, that will crop up constantly in discussions on the mechanics of arches and vaults:

1. the tripartition of the arch enables two stable parts of the arch and one unstable central part, in which the collapse mechanism is triggered off, to be identified.
2. The central part does not form a single body delimited by the joint planes that separate the stable part from the unstable part; instead, it is separated into two parts along the keystone line.
3. The two central parts, split up as stated, do not slide along the breaking joints but rotate around the intrados edges.

In Bernardino Baldi's argument it is possible to glimpse one of the reasons that must have given rise to the «empirical» rule suggesting that the intrados of the arch should be divided into three equal chords in order to be able to determine the thickness of the supporting piers (the so-called Derand's rule, not mentioned in the *Exercitationes*), while the complete absence of explicit considerations referred to friction between the parts can be noted. The collapse mechanism with rotation of the elements that are considered unstable, on the other hand, becomes very important. It must be noted that the conclusive statement, concerning the advantage of having thicker vaults, does not prevent Baldi from illustrating the building custom according to which it was suggested that vaults should be made lighter in the central part and that the space above the springers should be filled.

Philippe de La Hire was to dwell on this aspect many years later, tackling the problem of arches starting out from completely different assumptions.

Proposition 125 and the *Mémoire* of 27th October 1692

The *Proposition*, and consequently the *Traité* containing it, has always been analysed without referring in any way to the architectural context of the time, but if the reasons which induced La Hire to tackle this subject are considered in detail, it is possible to find many clues that necessarily change the point of view from which the work is seen. The arguments contained in the *Traité* lead clearly to the activity he carried on in the framework of the Paris *Académie d'Architecture* [5] and to his interest in *la coupe des pierres*, which are linked to the direct relationship he had with Desargues and with the unpublished work *Traité de la coupe des pierres*[6] (La Hire 1687–1690).

Proposition 125 must be viewed as the natural continuation of the discussions which ensued in the Academy following Leon Battista Alberti's *De re aedificatoria* reading. The minutes of these meetings are precise and circumstantial in this respect (Lemonnier 1911–1929).[7] On 20th October 1692 the academics (the *Compagnie*) commented the pages of Alberti's Treatise dealing with how thick the walls of round temples should be (Alberti 1485, book VII, chapter X): «Il dit que dans les temples ronds que l'on veut rendre fort solides, on doit donner aux murailles la moitié du demi diamètre intérieur du temple, ce que l'on a approuvé, en se réservant néantmoins d'en parler encore la première fois et de faire

quelques mémoires sur ces sortes de proportions». On 27th October, at the next session, La Hire read a *mémoire* to the *Compagnie* containing the observations developed by him on that same subject and which was to be commented on again at the time of the next session, held on 4th November.

The text attached to the minutes of 27th October resembles a draft of *Proposition 125*, however it also contains more structured thoughts, which consider in greater depth some aspects of the problem that were to be completely left out of the *Traité de mécanique*. Indeed, although the paper was still immature from the point of view of mechanics, was extremely explicit from that of construction. In it, La Hire situated the architectural problem and, above all, clarified his thoughts on the mechanical behaviour of arches and on the relevant *règles de l'art*. The *mémoire* submitted to the *Académie* and approved by the latter bears the title *Remarques sur l'époisseur qu'on doit donner aux pieds droits des voutes et aux murs des dômes ou voutes de four*, and started out from Alberti's proportions (La Hire 1692). La Hire proposed a comparison between the dimensions indicated by Alberti and those resulting from Derand's rule, and demonstrated that if the vault is full-centred the two indications coincide. He pointed out, however, that «cette règle ne peut estre fondée que sur quelques expériences», as is obvious if the piers considered are very high in relation to the size of the vault. Assuming, for instance, that the height of the columns is equal to one and a half times the diameter of the vault, the indications provided by that rule would turn out to be unacceptable.[8]

Before proceeding with his mechanical analysis, which has the intention of overcoming the fragile empirical nature of Derand's rule, La Hire expresses a decisive premise, which guides everything that follows it: if, in a stone vault, «tous les voussoirs ettoient tous poussez vers le centre de la voûte avec un mesme effort», then this vault would not thrust against the supporting piers «car la clef et les voussoirs d'en haut qui en sont proches ne feroient pas plus d'effort que s'ils estoient tous joints ensemble et s'ils ne faisoient qu'un mesme solides, dont il faudroit considérer l'effort comme celui d'une seule pierre toute droite pesant autant que tous les voussoirs ensemble et posée de niveau sur les pieds droits».

The observation expressed here is essential for understanding the first *mémoires* on the mechanics of arches. It heralds the theory developed later in the *Traité de mécanique* —to gauge the weight of the voussoirs so as to obtain «un mesme effort»— and the idea is pointed out that the keystone voussoir and those next to it have a special role in the statics of the arch, which distinguishes them from those close to the imposts. It is for this reason that La Hire goes so far as to state that if the «central» voussoirs were to consist of *une seule pierre*, the thrust would be cancelled out. A little further on this statement was to be altered slightly, however the conceptual reference remained the same and recalled that already expressed by La Hire at the meeting of 19th November 1688 about the *coupe des pierres* in «voûtes surbaissées».[9]

Proceeding in his analysis, with his attention focused on the single voussoirs, La Hire continues to refer to a 'non-thrusting' construction and the *monolithic model* constantly influences the reasoning he follows. This same consideration inspired the problem included in *Proposition 125*, that is to say, «donner une règle pour faire que les premiers voussoirs récompensent par leur pesanteur ou par leur charge l'effort de ceux qui sont vers la clef». Again in this case the attention was focused, as revealed in the title, «sur l'effort» produced by the voussoirs close to the keystone and not by all the voussoirs undifferentiatedly. The demonstration given in 1692 was altered in the *Traité* (1695), however not only did the basic idea remain the same on that occasion but, as we will see, it would also condition the *mémoire* of 1712, more than the text of *Proposition 125* did. In this latter work, furthermore, there was nothing to bring to mind a profound knowledge of a building site and of proper workmanship, with regard to which La Hire had simply reiterated well-known concepts.

The rules of proper workmanship do, on the contrary, play an essential part in the 1692 *mémoire*, in which an explanation according to «les principes de la Mécanique» is attempted. La Hire states that his mechanical considerations are clearly confirmed in site practices, in that the custom of loading the voussoirs close to the springers definitely confirms the validity of those observations, since experience had shown that this construction practice made the vaults more solid and safer.[10] At the end of the *mémoire*, this concept is reasserted in even stronger

terms. Since it had been demonstrated[11] that the keystone and the voussoirs that are close to it push against the abutments far more than do the other voussoirs, then it would be sufficient to remove the keystone and a few adjacent voussoirs for this thrust to be considerably lowered. The voussoirs close to the imposts would then require very little weight in order to withstand the pressure with which the others tend to press against them. Again in this case, on-site experience tended to confirm the theoretical reflection referred to the construction of dome: «Il est donc certain que l'usage d'ouvrir les dômes vers le milieu, comme pour y mettre une lanterne, soulage beaucoup la voûte et empesche l'effort des voussoirs à écarter les murs et piliers buttans».

The procedure that leads to the definition of the weight of the single voussoirs obviously comes up against the problem of the first voussoir of the impost, as was to be the case in *Proposition 125*. If the voussoirs are «infiniment polis, en sorte qu'ils peuvent glisser les uns sur les autres sans aucune difficulté», as is stated in the foreword, then there is no weight that could enable that voussoir to withstand the effort transmitted to it from those above. Once again, here, La Hire introduces a reflection drawn from his experience of construction: «C'est pourquoy on devroit arrester soigneusement ce voussoir avec le coussinet pour faire une bonne construction, si les inégalitez des pierres ne l'empeschoit de glisser sur le coussinet, et se sont aussi ces mesmes inégalitez qui récompensent en quelque façon les grandes charges qu'il faudroit donner à tous les voussoirs et surtout aux premiers».

What is today called friction came into La Hire's reflection, albeit in a still indeterminate manner, as the effect of the *inégalitez des pierres*. In this respect, it is important to note the difference as compared with *Proposition 125*, in which, on the other hand, «matiere qu'on met entre deux» was mentioned, stressing the importance of the presence of mortars and moving away from the world of *coupe des pierres*; if for no other reason then in order to respect the initial hypothesis regarding *infiniment polis* voussoirs, on which doubt could not be shed in a *Traité de mécanique* simply by means of a vague reference to experience.

In order to understand the context of the *mémoire* written in 1692 better, some considerations of a general nature should be added. The *monolithic model* from which La Hire started out had a clear precedent in stereotomy, and the assumption on which the *coupe des pierres* was based must be seen in the idea of a whole made up of *une seule pierre*. At the beginning of the *Traité de la coupe des pierres*, drafted five years earlier, La Hire had written the following, interpreting the entire stereotomic tradition in this way: «les ouvriers appellent la science du trait dans la coupe des pierres, celle qui enseigne à tailler et à former séparément plusieurs pierres, en telle sorte qu'étant jointes toutes ensemble dans l'ordre qui est leur convenable, elles ne composent qu'un massif qu'on peut considérer comme une seule pierre».[12]

The same concept was reiterated on other occasions, for example on 11th January 1694 on the subject of the drums of columns,[13] and the reflection on the subject of *comme une seule pierre* was subsequently to become the underlying theme of other research studies into the thrust of vaults.[14] The monolithic idea was a mainstay of stereotomic art, precisely because it was founded on the need to make up the all with the parts, to create the whole with the discreet. This monolithic nature could be realised ideally by perfecting the rules of workmanship but also by providing the additional solidity that was attributed to cramps or to the wedges, often dovetailed in shape, that were positioned between one voussoir and the next. The same solution is often also found in ancient and medieval architecture[15] and became a matter for discussion in treatises, as shown by De l'Orme's treatise (De l'Orme 1567).

La Hire himself discussed a similar issue in *Architecture civile* (La Hire 1698) and, only a few years after the *mémoire* analysed above, in his *Projet d'une nouvelle construction de murs de brique et de pierre de taille*, read and approved by the *Académie d'Architecture* on 14th September 1699, in which a new building system calling for the use of bricks was presented.[16] The same subject was treated by Jean Errard, who illustrated his *Premier livre des instruments mathematiques mechaniques* (Errard 1584) with very eloquent plates.

Without defining them, La Hire postulated two types of monolithicity, in addition to the type on which the *coupe des pierres* is based. These were based on the specific arrangement of the joints, and were an *acquired monolithicity*, featuring the use of cramps or wedges to connect the voussoirs to one another, and a *monolithicity secundum situm*,[17] due to the «liaison [. . .] de leur propre

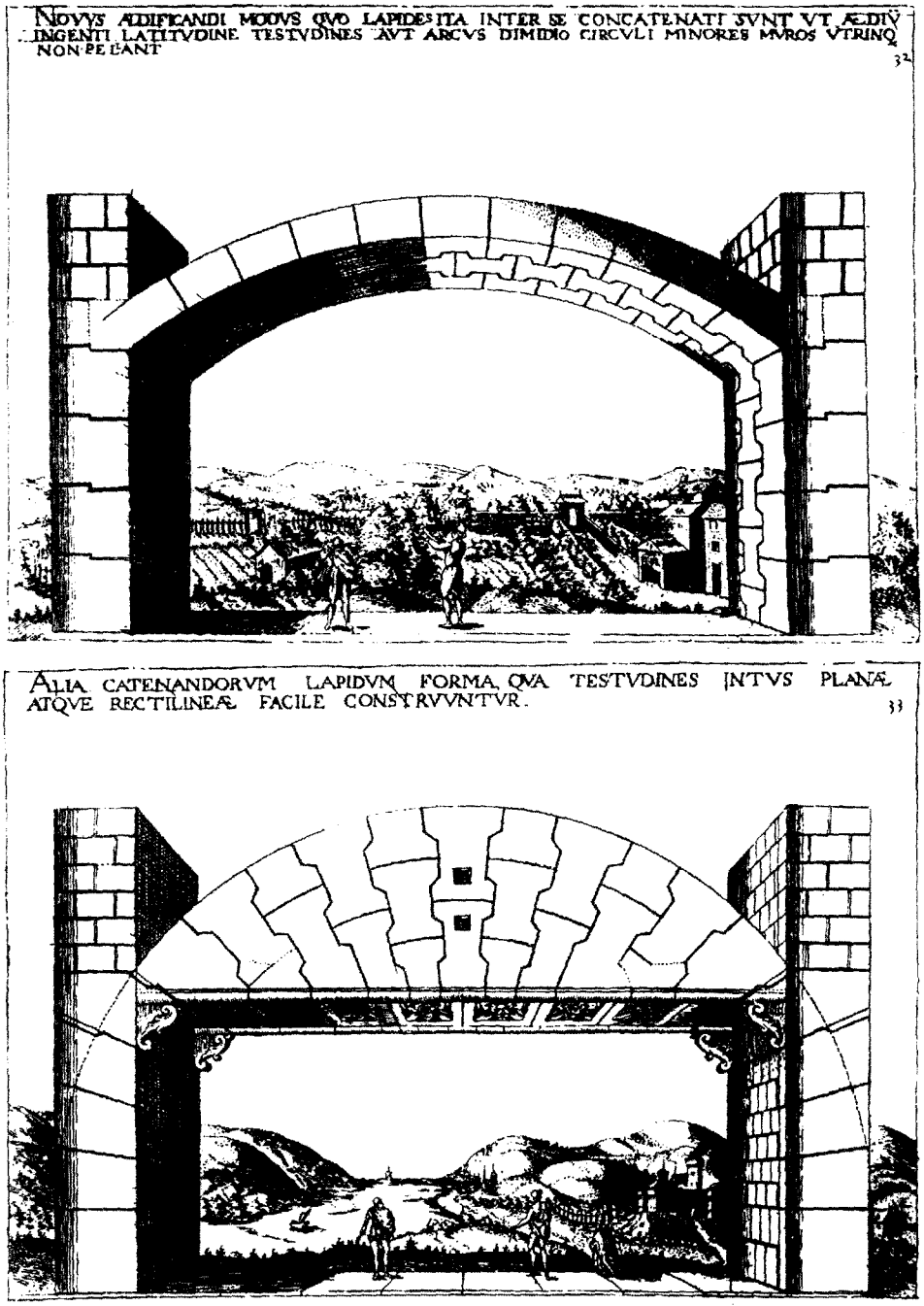

Figure 3
From (Errard 1584)

pésanteur»,[18] originating from on the theorem illustrated in 1692 and perfected in the *Traité de mécanique*. In both cases, the analysis was still developed in the framework of an investigation into the *force des voûtes,* and its stated purpose was to create a *non-thrusting* structure. In the first case the result depended on the effectiveness of the connections[19] and in the second on the «inegalitez des pierres», invoked in the 1692 *mémoire*, or on the «matiere qu'on met entre deux» mentioned in *Proposition 125*.

The considerations connected with the 1692 *mémoire* become even more eloquent on re-reading the one submitted to the *Académie des Sciences* on 27th February 1712. Again in this case there is good reason to believe that this text is similar to that discussed at the *Académie d'Architecture* during its session of 20th June 1711.[20] The same theoretical goal was considered from a different perspective, and the influence of the discussions held at the *Académie d'Architecture* could be felt yet again.

From 27th October 1692 to 27th February 1712

The 1712 text has been commented on extensively by many authors and rather than being necessary to re-examine it in full, it is sufficient here to pause to consider the initial hypothesis that conditioned the way in which it unfolds.

After noting that in architectural works the size of abutments varied from the excessive dimensions imposed by builders who were «moins hardis» and the insufficient dimensions due to the «trop hardis», La Hire states that «on remarque ordinairement que lorsque les pieds-droits d'une voûte sont trop foibles pour en soutenir la poussée, la voûte se fend vers le milieu entre son imposts et le milieu de la clef».[21] Starting out from this statement, apparently drawn from his experience and concerning the position of the breaking joint at an angle of 45° to the impost line (in the case of a semi-circular arch), La Hire deduces that «on peut supposer» that in the upper half of the semi-arches all the voussoirs are so well bonded to one another «qu'ils ne forment qu'une seule pierre». The whole subsequent reasoning for calculating how thick the direct bearing would have to be in order to guarantee the equilibrium of the arch is based on this supposition, taking the solidity of the foundations of these abutments for granted.

Commentators point out this hypothesis as an error, and do not analyse its contents.[22] The error, however, conceals much knowledge deserving of attention. It is obvious that the point from which the new theory started out coincided with the conclusions presented in the 1692 *mémoire*. The initial hypothesis is nothing other than a refined reproposition of what had already been written in this latter *mémoire* and, in particular, of the considerations on construction that had characterised it and that had disappeared in the *Traité de mécanique*: the voussoirs close to the keystone are the ones that produce the strongest thrust while those close to the imposts are integral with the supporting piers, also because of the «inegalitez des pierres». For this reason, La Hire had written that the central parts of domes could be made lighter in order to reduce the thrust on the walls. We should add that the load on the springers, which was recommended on that occasion, could give even greater reason to believe that a vault loaded in this way, in accordance with the rules of proper workmanship, would fail first of all at the point marking the upper boundary of the extrados area which was filled in.

Other references, however, came into the reasoning suggested by La Hire, although not in an explicit manner. In the *Académie*'s reports, the problem of the thrust of vaults was frequently associated with that of the thrust of soil and the correspondence between these two research areas was to be reaffirmed subsequently elsewhere, for example by Pierre Couplet at the beginning of his essay *De la poussée des voûtes*, read to the *Académie des Sciences* on 9th February 1729.[23] This link was considered entirely natural, and Pierre Bullet, in particular, had dwelt on these two aspects. As far back as 1686 he had treated the thrust of soil at the *Académie d'Architecture*[24] and on this topic he had later proposed his own theory in *Architecture pratique* (Bullet 1691), presented to the *Compagnie* on several different occasions. In this text it was explained that the angle of 60° must be considered the angle of natural slope, but also that it is preferable in calculations to refer to an angle of 45°, «pour tenir sur cela le chemin le plus seur» (Bullet 1691, 171). It does not seem improbable that La Hire would have taken this into account with reference to the possibility of the voussoirs remaining in equilibrium on the surface of the joint.

A second indication must be added to this, and it comes from a field of investigation that, although it

apparently has little to do with the *poussée des voûtes*, is actually connected directly with the topic at issue. In 1699 Guillaume Amontons had submitted a project for a «Moulin a feu»[25] to the *Académie des Sciences*, and had pointed out, with reference to this new invention, the problems of friction that condition the movements of this type of equipment. One observation, in particular, had baffled the academics, as it was in apparent contradiction with common sense (Amontons 1699a, 166): «Par ces experiences on peut remarquer, en passant, que c'est une erreur de croire, que les frottemens dans les machines augmentent ou diminuent à proportions que les parties qui frottent, ont plus ou moins d'étendue, et que la roue par exemple d'un moulin tourne d'autant plus facilement, que ses tourillons ont moins de longueur, ce qui d'ailleurs est une mauvaise construction, à cause qu'ils mangent incontinent les boëtes dans quoi ils tournent».

A lively debate on the truthfulness of this statement had immediately ensued at the *Académie,* and La Hire, who was an authoritative member of the *Académie des Sciences*, had carried out several experiments in order to clarify the terms of the issue. To this end he had conducted tests with samples of wood and marble, going so far as to confirm the independence of friction from the size of the area of contact and to explain those cases which could not be referred back to Amontons's intuition (*Histoire de l'Académie* 1699, 128–134). La Hire was therefore well aware of the phenomena arising out of *frottement*, that is to say out of the *inegalitez des pierres* of which he spoke in his *mémoire* in 1692 and which Amontons was to analyse, using exactly the same terms, in 1699.[26]

If we add to the *mémoire* of 1712 the references to the *coupe des pierres*, to the problems connected with the thrust of soil and to those arising out of friction, La Hire's hypothesis becomes easily comprehensible. It can be believed that the breaking point of an arch is located in the proximity of the angle beyond which friction is no longer capable of ensuring equilibrium (as had already been suggested in the *Remarques sur l'épaisseur*) and that this is at approximately 45° to the line defined by the imposts.

This interpretation was confirmed in full by the assertions of Amédée François Frézier, who had attended La Hire's lessons at the *Académie d'Architecture*, in his *Traité de stéréotomie*. In this treatise he wrote that, thanks to friction, the voussoirs of the vault do not slide over each other until an angle of about 22° «et même jusqu'à 25. dégrez» is reached. He also adds that even beyond this angle, up to 45°, they produce very little thrust, «puisque ce n'est qu'à cette hauteur que les Voutes se fendent». (Frézier 1737–1739, 3: 397). This is consistent with what La Hire had asserted in 1692 and reiterated in 1712.

The myth of Galileo

There is a profound difference between the mechanical reasoning followed by La Hire and the approach suggested by Baldi, and this is explained at least partly by the cultural contexts from which they originated. On the one hand there were the world of construction of the *architecture à la française* and the experimental research promoted by the *Académie des Sciences* in Paris, and on the other mechanics in the tradition of Aristotle and the overview of Italian brickwork architecture, in which *coupe des pierres* never played a leading role comparable to the one it had in France.

In La Hire's analysis, the study of the effect leading to an analysis of the cause was still influenced by a steretomic approach, which indicated the pathway to be followed: the stones at the top, comprised between the two breaking joints, behave like a single *voussoir* and the kinematics underlying the interpretation of this concern the large keystone voussoir which pushes against the springers of the vault, as already described in the foreword to the 1692 *mémoire*. The mechanism being analysed is that of the wedge, on which the whole issue of *coupe des pierres* rests, albeit without drawing any strict mechanical consequences. It is highly probable, therefore, that the error with regard to the collapse mechanism was caused not only by the fact that «on remarque ordinairement» as suggested by La Hire, but also by the stereotomic principle according to which a well-built vault would behave «comme une seule pierre». Having verified break in a particular joint, it was easy to reiterate the principle, provided of course the vaults were built with proper workmanship, possibly with the arrangements described by De l'Orme and discussed by the *Académie d'Architecture*. The prejudice referred to the monolithic nature of the central part of the arch,

which in Jacques Heyman's essay had already been indicated with lightning intuition as the «'monolithic' approach» (Heyman 1976, 30), thus corresponds in full. On viewing the collapse mechanism, it is possible to speak with good reason of a *monolithic paradigm*, handed down from the *coupe des pierres*, with an implicit but probably decisive influence of the research being conducted at that time into the thrust of soil and into the role of friction.

La Hire's reasoning, frequently described as naïve and summary, was in actual fact perfectly pertinent to the scientific and technical context in which he worked and, above all, it was consistent with the world of construction that La Hire was required to consider in the framework of the *Académie d'Architecture*. The arches and the vaults he imagined in the abstraction of the *art du trait*, but had also observed in the field during his many travels all over France, corresponded adequately to this model. The lever and the wedge, considered «simple machines» that become the rules of grammar for mechanical interpretation, were grafted onto a conceptual context with which La Hire was particularly familiar. The context in question highlighted the limits of a strictly mechanical approach and, at the same time, revealed the ties to that *art de bâtir* which it was, in any case, necessary to confront. *Acquired monolithicity* and *monolithicity secundum situm* were an unmistakable sign of this *internal* interesse in stereotomy, and the activity carried on at the *Académie d'Architecture* appears to be something more than a mere pretext.

In the collapse mechanism proposed by Baldi, on the other hand, the presence of studies of a different nature and an ill-concealed discomfort, shared moreover by La Hire, can be felt on linking the principles of mechanics known at the time with on-site experience. The latter is mentioned explicitly and illustrated with pictures in the *Exercitationes*, which can hardly be accused of an exclusive preference for the theoretical component of the problem. This was, on the other hand, to occur subsequently (markedly during the course of the 19th century). The difficulty lies, rather, in finding a way to reconcile the mechanical situation described by the principle of the lever and by the *scientia de ponderibus* with the situation brought to the forefront by the arch, imposing compliance with analytical procedures that are today part of the basics of mechanics applied to construction but which at that time had still to be described.

The absence of considerations on the theory of arches in Galileo's *Discorsi e dimostrazioni matematiche* —although he definitely recalled the *Quaestio XVI*— and the limits of his analysis of the mechanical behaviour of a cantilever beam —in which the curvature of the inflexed beam and, therefore, the preconditions for the analogy between beam theory and that of elastic curves are neglected— highlight a technical and scientific context *Before 1695* rich in subtleties that could hold many historiographic surprises in store. Researchers interested in the relationship *between mechanics and architecture* have the task of accustoming their gaze to the dazzling light of the myth of Galileo and of setting to work to re-write some parts of the *Histories* that up to now have been considered as *Reference Works*.

Notes

1. See the treatises mentioned in the references listed in (Becchi and Foce 2002). In (Benvenuto 1991) attention is drawn to some interesting pages by Honoré Fabri (Fabri 1669), however on the whole this approach is shared. Indeed, chapter 10, *First Theories about the Statics of Arches and Domes* (Benvenuto 1991, 2: 321–348), opens with an analysis of de La Hire's *Traité de mécanique*.
2. As yet there has been no thorough study of Leonardo's manuscripts dealing with this issue. Leonardo's writings are always viewed in a rather impromptu and non-systematic manner, as if his research into a given topic mirrored the fragmentary and asystematic nature of the subject-matter of the study.
3. For a fuller comment on de La Hire's *mémoire* see (Becchi 2002). The Author is currently working on a study of Bernardino Baldi's *Exercitationes*, to be published shortly with the title Quaestio XVI. *Dai* Mechanica *aristotelici alla meccanica per l'architettura: il contributo di Bernardino Baldi*.
4. «Why are pieces of timber weaker the longer they are, and why do they bend more easily when raised; even if the short piece is for instance two cubits and light, while the long piece of a hundred cubits is thick?»
5. De La Hire became a member of the *Académie d'Architecture* on 7th January 1687, to replace François Blondel, who had died in the previous year.
6. There are at least five copies of the manuscript, at the *Bibliothèque de l'Institut* and at the *Bibliothèque de l'École Nationale des Ponts et Chaussées* (this latter library has two copies of the manuscript) in Paris, at the

Bibliothèque municipale in Rennes and at the *Bibliothèque municipale* in Langres. See (Becchi and Foce 2002).

7. From now on only the date of the minutes will be mentioned.
8. (La Hire 1692): «Cette règle ne peut estre fondée que sur quelques expériences, car on ne peut pas assurer qu'un arc de pierre par exemple de 12 toises de diamètre, dont les pieds droits seront de 3 toises d'époisseur, soit si ferme qu'il n'ait pas besoin de piliers buttans ou de culée pour l'entretenir. Au contraire, il est très certain que les voussoirs feront toujours assez d'effort pour écarter les pieds droits, surtout s'ils sont d'une hauteur considérable comme d'une fois et demie le diamètre de la voûte».
9. «M. de la Hire a apporté à la Compagnie une démonstration dans laquelle il fait voir que, dans les voûtes surbaissées, la clef et les autres voussoirs qui en sont proches font plus d'effort pour escarter les premiers voussoirs que ceux-cy n'ont de force pour y résister, ce qui se prouve par la proprieté du coin, qui est plus aigu dans la clef et dans les voussoirs qui lui sont proches que dans les autres. Il y auroit plus de solidité si tous les joints de lit tendoient au centre de l'ovale qui forme le cintre surbaissé; mais cela n'est pas si agréable à la veue, cependant on est obligé de tomber en ce cas en plusieurs rencontres». This statement is significant, since it reverts to a consideration that had already been introduced in (La Hire 1687–1690) and makes it more explicit in the direction that was to be developed four years later (La Hire 1692).
10. (La Hire 1692): «c'est pourquoy les voûtes donts les reins sont bien remplis ont toujours plus de solidité et de fermeté que les autres».
11. (La Hire 1692): «On peut voir par la proportion que je viens de trouver que la clef et les voussoirs qui en sont proches font un bien plus grand effort dans une voûte pour écarter les pieds droits que les autres qui sont vers le coussinet».
12. (La Hire 1687–1690), sheet 1. Passage also contained in (Pérouse de Montclos 1982, 85), but with a transcription error : «pièce» instead of «pierre».
13. See minutes of 11th January 1694, *mémoire* bearing the title *Nouvelle manière de former des colonnes par tambours*: «Je crois que la meilleure de toutes les manières dont on puisse se servir pour poser les pierres, c'est de frotter les lits les une contre les autres avec un peu de grés et d'eau et de les arrester ensuite à la place où ils doivent demeurer. Car, par ce moyen, ces pierres se touchant exactement par leurs lits et ne pouvant pas s'approcher plus d'un costé que d'autre, ne forment que comme une seule pierre, et les arrestes des joints ne sçauroient s'éclater pour quelque charge qu'on élève au dessus».
14. See (Frézier 1737–1739, 3: 382): «M. Danyzy fit ensuite voir par une expérience que plus la clef est large moins la poussée de la Voute est grande: car si l'on substitue à trois ou à plusieurs Voussoirs une seule clef qui occupe tout l'intervale qu'ils remplissoient, et qui soit égale à leur somme, on verra que la Voute qui n'auroit pû se soûtenir après avoir un peu diminué de la force des piédroits, se soûtiendra cependant encore lorsqu'on y aura mis cette clef, quoiqu'elle soit aussi pésante que l'étoient les Voussoirs, non dans l'état d'équilibre, mais lorsqu'ils surpassoient la résistance des piédroits. D'ou l'on tire naturellement une conséquence que nous avons établie ci-devant pour une chose constante, que si la Voute étoit toute d'une pièce, la poussée déviendroit nulle». See also (Danyzy 1732, 52) and (Cosseron de Villenoisy 1869).
15. On this topic see (Reveyron 1996).
16. In the *mémoire* it is stated that: «Les anciens architectes ont pris de très grands soins pour lier toutes les pierres qui formoient les gros murs des édifices considérables, et nous voyons dans ceux qui sont batis de gros quartiers de marbre que toutes les pierres sont attachées les unes aux autres avec des clous et des harpons de bronze. Aussi ces édifices, après un grand nombres de siècles, sont aussi entiers que s'ils étoient nouvellement construits». (see attachment to the minutes of 14th September 1699).
17. We suggest this definition with obvious reference to the *gravitas secundum situm* described by Jordanus de Nemore.
18. According to the beautiful expressed used by Frézier. See (Frézier 1737–1739, 1: vii-viii): «Il faut en effet plus d'industrie qu'on pense pour que [les petites parties] soient [. . .] disposés de manière qu'elles se soutiennent en l'air, en s'appuyant réciproquement les unes sur les autres, sans autre liaison que celle de leur propre pésanteur».
19. There is a precise filiation, on which it is not possible to dwell here for reasons of space, between the *acquired monolithicity* investigated by de La Hire and the investigation of *linteaux armés*. On this latter subject, see (Saddy 1987) and (Middleton 1987).
20. The text of this *mémoire* is not attached to the minutes. See (Lemonnier 1911–1929).
21. (La Hire 1712, 69): «On remarque ordinairement que lorsque les pieds-droits d'une voûte sont trop foibles pour en soutenir la poussée, la voûte se fend vers le milieu entre son imposts et le milieu de la clef; c'est pourquoi on peut supposer que dans la moitié supérieure du demi-arc, tous les voussoirs sont si bien liés les uns aux autres, qu'ils ne forment que comme une seule pierre: et c'est sur cette supposition et sur la solidité de la fondation où les pieds-droits sont assis, que l'on établi la démonstration de la regle que nous trouverons dans la suite».

22. See the extensive references contained in (Becchi and Foce 2002). Only Jacques Heyman exhibits great caution; see, for example, (Heyman 1972, 82–84 and 168).
23. The *Académie* was to enlarge once again on these two issues on 13th March 1713, in the presence of Bullet and de La Hire, when treating «de la construction et de la poussée des voûtes et aussy de la construction des murs de terrasses». A similar correspondence was to be reiterated in the report drafted by the *Académie des Sciences* on Couplet's *mémoire* referred to above: «Aprés ce que M. Couplet a donné sur les Revêtements des Digues, Chaussées, &c. il étoit naturel qu'il pensât aux Voûtes, dont la Théorie doit dépendre des mêmes principes de Méchanique». See (*Histoire de l'Académie* 1729). It is also well known that C.A. Coulomb was to turn his mind to the same problems.
24. See 17th May 1686.
25. (*Histoire de l'Académie* 1699, 124–127). The *mémoire*, read by Amontons on 20th June 1699, is quoted in (Amontons 1699a).
26. (Amontons 1699a). We owe to Amontons a second, fundamental *mémoire* on the subject, read to the *Académie des Sciences* on 19th December 1699 (Amontons 1699b).

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