Helical masonry vaulted staircases are complex structural elements because of their geometry, technology and mechanical behaviour. These elements have been found in some historical buildings, and their *Firmitas* and *Venustas* wake the designer up nowadays.

The research aim is to analyse the masonry staircases built during the second half of 1500, and in particular these built by eminent architects and writers of treatise as Andrea di Pietro Dalla Gondola, called Palladio, and Jacopo Barozzi da Vignola.

The research of historical bibliographic sources is important and fundamental for investigating any historical architecture. Therefore, the architectural, technological and structural aspects have been considered simultaneously to analyse the structural behaviour of helical masonry vaulted staircases. The author have consulted treatises and handbooks of XVI c. and following, besides observing and surveying these element in situ.

The detailed analysis of these different aspects allows to formulate a static scheme of the structure, boundary conditions and the applied loading. Some architectural elements seem to have a main role in the staircase Statics, guaranteeing its structural behaviour in service conditions.

**INTRODUCTION**

The staircase has always been an important architectural element, both for symbolic meanings related to it and the difficulties due to its design and building. The staircase is a fascinating design theme for artists. It is a comparing standard, perhaps a way to compare themselves in terms of ability. During the Humanism, famous artists as Leonardo and Francesco di Giorgio Martini designed staircases full of symbolic meanings, whereas Alberti, in his treatise (Alberti 1485), emphasises that staircase is an architectural element which is difficult to be realised. The first monumental staircases date back to the beginning of XVI c. due to Michelangelo, Bramante, Antonio da Sangallo and others. Subsequently, the theme is investigated deeply and with curiosity as a «challenge», until the Seicento. During this century the staircase theme achieves the maximum inventiveness.

The staircase theme in historical architecture is extremely wide and complex, although the referring bibliography is limited. Therefore, the authors have turned their attention to the topic referring to a precise historical period, the second half of 1500, and to two architects which are the most important in Northern Italy, Palladio and Vignola, limiting the research to only one structural typology, the helical masonry vaulted staircase.

**THE STAIRCASES IN TREATISES**

**Since XV at XVI century**

The first treatises of Architecture divulge during the XV and XVI c. During this period, the «publicistica»
spread everywhere. Indication of staircase design, comfort, safety and collocation in building could be found in every Renaissance treatise of Architecture. Only the last Renaissance treatise includes indications of building procedures: Scamozzi’s treatise at the beginning of XVII century.

The Vitruvio’s treatise is the reference for architects during the Humanism and the Renaissance (Gambardella, 1993). However, the staircases are not taken into much consideration in this document; in fact, as Frà Giocondo Da Verona complained in edition of 1590, the rules for staircases design are missing. This is due to the fact that the staircase is not and architectonical element in the roman domus, whereas it is an imposing base in the public buildings, and often it becomes an architecture itself, as Serlio said (Serlio 1584).

Already in «De Re Aedificatoria» of Leon Battista Alberti (Alberti 1485), the staircase has an important role as useful element in the building but, at the same time, its presence makes the design difficult. The staircase is so necessary that «who would not like the staircases hinder, he avoids hindering the staircases», underlining that it is not possible to neglect the place devoted to them, although this could induce some difficulties to the designer in term of space organisation.

In Francesco di Giorgio Martini’s treatise, titled «Architettura civile e militare», the winding staircase is represented (Gambardella, 1993). It is located in a tower, as only entrance to a second defensive tower; in this way, whereas the enemy cover the long climb, the soldiers organise the defence. The staircase becomes tool of war and life.

If, in Cataneo, the courtyard is the centre of building and the staircases regulate only the space organisation (Barozzi and Cataneo 1560), Alvise Cornaro in 1556 wrote (Barozzi and Cataneo 1560): «the designer has to give space to staircases and not hinder them, because hindered, they hinder», following the same principle of Alberti.

In Sebastiano Serlio’s treatise (Serlio 1584), «I sette libri dell’architettura», the author declares that the helical staircases are «arduous structures to be built so that who is not able to design the traditional ones he should not even try to design those are more complex». The staircase is considered a difficult element which is understandable only to expert designer. He inserts the staircase in some perspective studies and he tries to furnish an handbook for designer. He takes again the theme in volume III, dedicated to Roman antiquities, where he writes about the staircase -building: Colosseo. In the analysis of Roman architectures, Serlio emphasises well made staircases, examples have to be followed by the contemporary architects. The Bramante’s helical staircase stands out among the quotations; it is the only modern architecture, which has to be admired and studied. It is pointed out that the stairwell is designed carefully in every plate; evidently, the architect considered the staircase an important and difficult element in the architecture.

Giorgio Vasari, in «Introduzione all’Architettura» having an informative, teaching and technical aim, furnishes brief and shorted description, but extremely important for understanding how this architectural element is considered at this time. The staircases used by people in public building have to be comfortable, not steep, roomy, bright; they have to be magnificent. Besides, the author recommends that the designer finds an appropriate place for staircases because «this element is difficult to be located in the building» (Barozzi and Cataneo 1560). The staircase is almost more important than main rooms; it is a needed structural element having also representative role, like a beauty gauge of construction so that «a lot of people see the staircase and not the remainder of the building» (Barozzi and Cataneo 1560).

The staircases realised by eminent architects, as Bramante, Michelangelo and Philbert De l’Orme, are many. The last one gives a definition of staircase in his manuscript «Le troisième livre de l’architecture», 1567: it is like the «beating heart» of building (Gambardella 1993). The main example that has influenced many architects is the double staircase of Chambord (1515–1525), built by Domenico da Cortone for the will of Francesco I, probably following Leonardo da Vinci’s project. In fact, Vignola, who has been in France for a long period, would be influenced greatly, whereas Andrea Palladio has drawn and described it, figure 1, in the first of his «I Quattro Libri dell’Architettura», with that of Bramante and some Roman antiquities. In Palladio’s treatise, in the same chapter, many detailed figures and many indications could be found concerning the staircases position in the building; besides, he catalogues staircase typologies that could be built. Technical indications about their built are not in this treatise.
The first who gives this sort of indications is Vincenzo Scamozzi. In his treatise, «L’idea dell’architettura universale» (Scamozzi 1615), he is interested in architectural, technical and building aspects of staircases. He defines the right position of staircase in the building and different typologies in Libro II, the ornaments in Libro VI and the building indications in Libro VIII. He is the first who has distinguished the service staircases from the main ones explicitly. Besides, he indicated which material has to be preferred, in particular he disapproves wood due to its inflammability. The chapter about building indications is not clear and it is lacking in building details; however it is the most complete treatise up to this century.

The lack in building indications in treatises up to the XVII c., previously cited, is probably due to the architect who relies on mason ability. The architect is interested in formal and figurative aspects, delegating the staircase realisation to the mason, who knows well materials, building technologies due to his builder’s yard experience.

Modern centuries

It is possible to find more information about building technologies looking for in the treatises close to XIX c., when Breymann dedicated a book of his treatise to staircases (Breymann 1853). In this book, for the first time, a distinction of staircases is proposed in two categories: staircase «a collo» (it leans on continuum support, as piers, columns, walls, ecc.) and staircase «a volo» (it leans on support only at the beginning and the end of the flight), differing for building technology. This distinction has been used widely and it is also in Daniele Donghi’s handbook (Donghi 1923). These two treatises are still today the main references about technology in historic architectures.

Another distinction could be done in term of material for staircase building: stone staircase and masonry staircase. The manuscripts on stone staircases are numerous. The main reference is «Encyclopedie Medievale», wrote by Eugène Emanuel Viollet Le Duc, where stone brick staircases, which are widespread in Northen Europe, are precisely described and drawn (Viollet Le Duc 1868). According to Breymann, the choice to build a masonry staircase, rather than a stone one, has simply an architectural reason because it is against any economical and practical aspiration. The professor of Stuttgart reports the Northen Europe building culture in his handbook. The clay brick has a secondary role due to plenty of stone and its traditional working, especially about its cutting. Instead, in country like Emilia Romagna, bricks are the main employed material for every buildings, due to the lack in stone and the plenty of clay. The difficulty related to bricks employment is undeniable, especially for vaulted structures which are built assembling many bricks compared to a monolithic beam. It is singular that Palladio works in a country where he could choose between clay brick and stone, therefore his choice to design a vaulted staircase has an architectural reason. In particular, according to Donghi, the safety of...
masonry vaulted staircase, which is included in «a collo» staircase, depend on boundary continuity.

The vaulted masonry staircases could be built with the following technologies:

- **Staircase on vaulted beam**: is built on flat arch or plate-bande at each riser. In this way, only one wooden centering could be used, moving it following the staircase building. This technology is used for service staircases with small span, without architectural value; the vault intrados is covered with plastered «cannucciato» or similar.

- **Staircase on flying vault**: is built on flying vaults «a collo d’oca». The landings have to be built with particular care because they support the flights as abutments. The vault thickness is 14±17 cm or more (multiring), depending on vault span. The staircases on flying vault could be symmetrical or asymmetrical, called respectively «a collo d’oca» or «zoppe».

- **Staircase on barrel vault**: Breymann says «the most simple staircases are those with linear flights and landings, that could be easily supported by barrel vaults» (Breymann 1853). However, it is possible to find some example in which the barrel vault has a curvilinear development as in figure 2.

Thick masonry walls need for this typology for carrying the abutments trust; the vault could be cylindrical or flat. The helical staircases on barrel vault are unusual for the complexity of their geometry and the accuracy needed for bricks disposition.

- **Staircase on Roman vault**: this typology of masonry vault is widespread in Southern Italy, especially at Rome. It is particularly important because it has been used by Vignola in his spectacular helical staircases. A first description of this typology could be found in Donghi’s and others handbooks, figure 3. «A structural solution of great interest could be the cantilever staircase on roman vaults, made up of three flying vaults, two middle vaulted landings and one main vaulted landing; the vaults are made of clay bricks with small dimensions, flat or head disposed. The flying vaults look like special vaults «a collo d’oca», carried by boundary masonry walls and the adjacent vaulted landings, where middle landings are one quarter of vaulted rip roof and the main landing is a barrel vault» (Macchia and Oggioni 1995).

Figure 2
Helical masonry vaulted staircase (Breymann, 1853)
The particular disposition of bricks guarantees the Statics of vault, which looks like half arch. Besides, Vignola understood that, removing landing, the cantilever structure could be continuum and follow the climb without helical staircase breaks. The Vignola’s helical staircase differs from the traditional one.

PALLADIO AND VIGNOLA COMPARED ABOUT STAIRCASE THEME

During the second half of XVI c., Andrea Palladio, probably the most famous architect, turns his attention to staircase theme, avoiding complex structures and hiding them in massive masonry walls. However, he realises, in villas, the only masonry vaulted staircases in Veneto, during this period. Contemporary in Emilia, Jacopo Barozzi da Vignola designs his wonderful staircases, which are architectural and technological masterpieces for their hazard shapes obtained with a wise employment of materials and geometrical rules. The artists turns their attention to this theme in different way even if using the same materials and architectural shapes.

The staircase in Palladio

The role of staircase in Palladio’s architecture (1508–1580) is a debate theme for many years. Palladio creates an architectural system which is described in his illustrated treatise «I Quattro Libri dell’Architettura», which is divulged as far as the Northern Europe. The authors analyse Palladio as villa designer, the building typology that has given him worldwide fame.

Veneto Villa is an architectural typology which is defined for the first time by Palladio around the middle of XVI c. in book II. Generally, Palladio has
two client typology: noblemen of Venice (from Vicenza and Padova), well-to-do and politically important, and noblemen of «terra ferma» (from Vicenza e Verona). The client authority is reflected in building: the social status of client, not his economical power, defines the residence appearance. Palladio indicates the criteria which guide the design of noblemen, lawyers, merchants houses. Generally, the villas for Venice noblemen have two floors with pronao at the front, whereas the others villas are developed in width with only one columns row (Conforti and Tuttle 2001; Puppi 1995).

The staircases designed by Palladio are made of stone and the stairs are fixed at the lateral walls; they are located in small rooms, far from the main rooms. The unusual masonry staircases are also of limited dimensions, above all, they are placed in narrow rooms cut out between a room and the others. The flights are rectilinear in many case, however the staircase plan is triangular in some buildings as in Rotonda (Blanc 1996).

In chapter XXVIII, Libro I, «Delle scale e varie maniere di quelle . . . » many pages are dedicated to staircases, with detailed drawings about magnificent staircases. Particularly, Palladio dwells on the staircase position in the building, as Alberti has already done. He asserts that the right position of it could make the whole house more magnificent, emphasising the most beautiful rooms and hiding the smallest and less designed rooms.

Figure 4
Plan and prospect of Pisani Villa (Palladio 1570).
Although these indications, it seems that Palladio does not practise his teachings: in many villas, the staircases are located in small and hidden rooms, difficult to be used. Also when the staircase has wide dimensions, they were enclosed in thick walls and they are not part of house architecture, because they were enclosed in specific rooms which are not visible from the main rooms, figure 4.

The staircase becomes a secondary element, necessary for enjoying the building but not for its architectural characterisation. Palladio does not employ the architectural code of this period, as concave and convex staircase, placing his staircases in small towers hidden in the load bearing structures. Palladio reduces the staircase to a joint element between one floor and the other. The staircase as architectural element appears only outside: magnificent base which raises the building from ground, recovering the staircase theme of religious architecture (Chastel 1965).

Although that, Palladio realises three particular staircases for architectural shape and technology. These staircases are located in three villas belonged to Venice noblemen from 1550 to 1560: villa Pisani, now Placco (1552), Montagnana (Pd); villa Cornaro, now Gable (1553), Piombino Dese (Pd); villa Foscari «La Malcontenta» (1559-60), Mira (Ve).

These buildings are the only that have helical masonry vaulted staircases. They are a particular typology of villa, following Ackermann’s indication (De Fusco 1981), having two floors with two columns rows, isolated without rural buildings around. The villas in Montagnana and Mira are not exactly villas. The first one is called by numerous authors Palace due to the neighbourhood of Montagnana, the second one is realised as holidays palace close to Venice, without farm role. Pisani and Cornaro Villas have a four columns entrance, two helical staircases on the back for jointing the two superimposed loggias towards the garden (Prinz 1969).

The staircase in Vignola

The Jacopo Barozzi da Vignola’s work (1507–1573) is characterised by staircase as fundamental element in architectural house composition. Loukomski said that Vignola uses «staircases, obelisks, frescos and porticos, placing them to do better views» (Loukomski 1927). The architect lives in Rome, Emilia and France. His young training is done as painter in Serlio’s school, according to some researchers. During his training he works on architecture, sculpture and hydraulic engineering. Undoubtedly, Vignola is a careful observer of architectures realised by his contemporaries and he has carried out important architectural and structural starting points. In particular it is possible to find the influence of Baldassarre Peruzzi and Giulio Romano (Orazi 1982; Walcher Casotti 1960; Tuttle et al. 2002).

With the purpose to compare Palladio and Vignola about the same theme, the authors consider the staircase realised in private buildings, in particular in Country Palace which has different characteristics in Emilia than in Veneto villas. The country palace is designed and realised with the same criteria than the city palace; the palace is conceived like «a piece of city granted to countryside» (Cuppini and Matteucci, 1967) and its architecture is conceived apart from the adjacent agricultural buildings. The bigness and magnificence of building is proportional to the economical power of client, and not related to his social status (Scannavini 1998).

The staircases designed by Vignola are always spectacular and fruit of careful investigations. In fact, Vignola takes care of their design and dimension evaluation, choosing complex shapes and hazarded structural solutions. His staircases are calculated empirically without mathematical tools. It is observed from his drawings that one building cross section goes through the staircase always; in many project, there are different cross section of the same staircase or graphical analysis on the stairs dimension, as in Cervini Villa, figure 5.

The absolute knowledge of building technologies, materials, geometrical roles is evident in Vignola’s masterpiece. Vignola knows that the staircase is a complex architectural element and, probably for this reason, he is fascinated by it.

The authors concentrate on the staircase of Boncompagni Palace, at Vignola (Mo). The palace attribution to Vignola is doubtful, whereas the staircase design looks to be his hand; instead, the realization of masterpiece is due probably to Bartolomeo Tristano (Tutle et al. 2002). The author reaches at this project, perfect synthesis of shape and dimension, after long work experience. A chronology
of Vignola’s staircases is reported in table 1. It could be not complete and exhaustive from historical and architectural point of view, but it is useful for demonstrating the experience path followed by Vignola.

**Comparison on architectural, technological and building points of view**

Palladio and Vignola turn their attention to indoor staircases in different ways. In Vignola the staircase is a spectacular element for its architecture and structure, figure 6, whereas in Palladio it is hidden in massive masonry walls, figure 7.

Really, both of them contain the staircases in turrets, defiladed respect to the others rooms, and prefer elliptical plan (Blanc 1996). While Palladio’s staircases have a utilitarian function, to joint different floors, Vignola’s staircases have a aesthetic function in building architecture and the whole staircase is seen by the user. Nevertheless, an aesthetic purpose could be found also in Palladio. Who covers the staircase leaves back a room, then he appears suddenly at another room, which is different or analogous, for looking at it from a different point of view. The staircase is the tool for showing the most beautiful parts of house.

Both the authors design unusual structures for shape and structural behaviour. From the technological point of view, either Palladio or Vignola employed clay brick masonry for building their masterpieces. This material has a low tensile strength and good compressive strength. Palladio chooses a material employed for realising structural elements which are usually compressed, rather than using monolithic stone blocks fixed at one or both ends. Besides, the masonry choice entails the design of a vaulted structure, which looks to be itself bearing for the bricks disposition, as in Pisani and Cornaro villas. The vault span is limited (about 1.60-1.70 m) and it is fixed to boundary walls and central pier.
Table 1. Chronology of Vignola's staircases

<table>
<thead>
<tr>
<th>Time</th>
<th>Masterpiece</th>
<th>Typology</th>
</tr>
</thead>
<tbody>
<tr>
<td>1530–1540</td>
<td>Isolani Palace (Minerbio)</td>
<td>Wooden helical staircase</td>
</tr>
<tr>
<td></td>
<td>Boncompagni Palace (Bologna)</td>
<td>Masonry helical staircase</td>
</tr>
<tr>
<td></td>
<td>Cervini Villa (Montepulciano)</td>
<td>Masonry vaulted staircase on triangular plan</td>
</tr>
<tr>
<td></td>
<td>Ninfeo in Giulia Villa (Rome)</td>
<td>Stone staircases</td>
</tr>
<tr>
<td>1550–1560</td>
<td>Farnese Palace (Caprarola)</td>
<td>Masonry vaulted staircase «a collo» with columns support on one side</td>
</tr>
<tr>
<td></td>
<td>Farnese Palace (Piacenza)</td>
<td>Small helical staircases and traditional ones</td>
</tr>
<tr>
<td>1560–1568</td>
<td>Isolani Palace (Bologna)</td>
<td>Masonry helical vaulted staircase with roman vault boundary conditions</td>
</tr>
<tr>
<td></td>
<td>Boncompagni Palace (Vignola)</td>
<td>Masonry helical vaulted staircase with roman vault along the flights without landing</td>
</tr>
<tr>
<td></td>
<td>Caprarola (hospital, Pozziello Palace, playing court)</td>
<td>Numerous helical staircases</td>
</tr>
<tr>
<td>After 1568</td>
<td>Fortification walls of town for Savelli family (Castel Gandolfo)</td>
<td>A magnificent and two small masonry vaulted staircases enclosed in fortification walls</td>
</tr>
<tr>
<td></td>
<td>Town Hall of Castro (Viterbo)</td>
<td>Stone helical staircase (outlived the building after earthquake).</td>
</tr>
</tbody>
</table>

Instead in Foscari villa, the structure is composed of a system of flat arches, like plate-bandes, under each stair, jointed by flat bricks of stairs. The intrados is realised with plastered «cannucciato» vault shaped.

Vignola, who is used to build with masonry because in Emilia the stone is an unusual materials, distinguishes himself inventing a daring building technology for realising scenic and spectacular structures, able to resist static loading (dead load and service load), and also seismic loading (helical staircase in Town Hall of Castro). A particular technology is noted in two staircases: the first in Isolani Palace, at Bologna, and the second in Boncompagni Palace, at Vignola. These structures are designed starting from the roman staircase typology, modifying some building aspects. In fact, Vignola eliminates the middle landings designing a single continuum helical flight. In Isolani Palace, the staircase starts from the underground floor with a barrel vault fixed at boundary walls and at central pier; from the ground floor, the vault is configured as roman vault and there is not the central support, leaving place to stair-well and masonry parapet. The flight stops at the only final landing, which is realised like a barrel vault following the traditional roman staircase typology. The staircase in Boncompagni Palace seems to be an evolution of the first one. In fact, there is not final landing and the staircase is a continuum helical flight with roman vault.

Comparing both architects about materials for realising their staircases, it is evident that the mortar is different, although both of them used masonry. Palladio uses lime mortar, come from Veneto quarries. Vignola uses limes from Emilia, mixed with plaster (Marinelli and Scarpellini 1992). The plaster modifies the mortar characteristics giving expansive property; so that the bricks are compressed after the mortar curing.

From the technical point of view, meaning the quantity evaluation of mechanical forces, both the architects calculate structure dimensions with geometrical tools. The staircase realisation is the fruit of experiences carried out in situ and observing the past architectures. On the other hand they have not
scientific knowledge about material strength and structural behaviour of architectural elements.

**STRUCTURAL BEHAVIOUR OF MASONRY VAULTED STAIRCASES IN PALLADIO AND VIGNOLA**

The following remarks are based on direct observation of these structures, recording their geometry, masonry configuration and damages that have been in the past or it is going on. The structure considered in this research have not shown any damage, visible or historically documented. Staircases of Pisani villa and Boncompagni Palace are reported as examples of Palladio and Vignola respectively.

**Palladio in Pisani Villa, Montagnana (Pd)**

The staircase amounts to a particular vaulted structure, which is developed rotating a circular arch around the central pier; the plan is elliptical and the boundary walls are massive. It is plausible that the vaulted structure and boundary walls are realised contemporary. The vault intrados is almost everywhere...
like a round arch, besides some variations of curvature could induce to think that the vault has been realised without wooden centering. The bricks are tilted respect to the vertical direction toward the flight starting, about an angle so that the following ring leans on the previous one, figure 8.

The boundary conditions look to be fixed ends. The modest dimensions of these vaulted structure (span of 170 cm, thickness at crown 15 cm) suggest two different static scheme, figure 9: beam with variable cross section or round arch, both fixed at ends.

The analysis carried out for both of them has done for a generic stair (depth of 34 cm), loaded by a uniform dead load. A linear elastic analysis is carried on because the structure is not damaged. The stresses evaluated for the two configuration are less than the ultimate masonry strength, table 2, considering a ultimate tensile stress of 0.20 N/mm$^2$ (Belluzzi O. 1994). The fixed arch scheme is the best to describe the structural behaviour of this typology of staircase. This remark is supported by the bricks disposition, the massive walls and the central pier which are able to receive the horizontal thrust at springings, and finally by the calculated stresses.

**Vignola in Boncompagni Palace, Vignola (Mo)**

The structural behaviour of this staircase is more complex and it could not be reduced to a bi-dimensional problem, because the structure amounts to an helical flight fixed at the boundary walls, with almost circular plan, and collaborating with the parapet which is helical and has a thin rectangular cross section.

The flight cross section is variable from a minimum of 15 cm at fixed joint and a maximum of 30 cm at the other end, toward the stair-well; it looks as a rib flat arch with a span of 200 cm. Some steel bars have been found along the flight, one each three/four stairs, and they are closed to the intrados. The roman masonry vault appears like a series of half skewed arches fixed at the crown and free at the springing; the brick layers are tangent to the boundary walls and perpendicular to the stair-well. The masonry parapet is jointed to the staircase structure; it is 100 cm high and 15 cm thick. The whole structure is fixed at the flight starting and hinged at the flight end. In fact, the flight that connects the underground and ground floors amounts to a masonry barrel vault supported by boundary walls and central pier: the system seems to be like staircase foundation. The flight structure is an helical vault which rises for two floors (about 12 m), doing 720° (2 turns) for rising from a floor and the other, figure 10.

The static scheme able to describe staircase structural behaviour has to consider the whole structure, that is the collaboration between roman vaulted flight, parapet and boundary masonry walls. It is possible to give different interpretations.

Firstly, a single stair is considered. Its static scheme is a cantilever with variable cross section, fixed at the boundary walls and elastically hinged at the other end (rotational and extensional hinges),

![Beam with variable cross section](image1)

![Round arch with constant cross section](image2)

Figure 9
Static scheme for Pisani Villa staircase
Table 2. Stresses in Pisani Villa staircase, considering fixed round arch and fixed beam static schemes

<table>
<thead>
<tr>
<th>x [mm]</th>
<th>M [N-mm]</th>
<th>J [mm²]</th>
<th>σ [N/mm²]</th>
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<td>2.41 10^5</td>
<td>9.56 10^6</td>
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<table>
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<th>N [N]</th>
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<td>-267.96 10^3</td>
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<td>0.034/0.091</td>
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<td>-232.58 10^3</td>
<td>0.112/-0.001</td>
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<td>-690.88 10^2</td>
<td>-208.41 10^3</td>
<td>0.136/0.034</td>
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<td>-608.01 10^2</td>
<td>-183.08 10^3</td>
<td>0.119/0.030</td>
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<tr>
<td>50</td>
<td>-334.07 10^2</td>
<td>-158.83 10^3</td>
<td>0.080/0.002</td>
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<td>14.27 10^2</td>
<td>-137.67 10^3</td>
<td>0.032/-0.036</td>
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<td>70</td>
<td>338.57 10^2</td>
<td>-121.29 10^3</td>
<td>-0.012/-0.071</td>
</tr>
<tr>
<td>80</td>
<td>564.21 10^2</td>
<td>-110.94 10^3</td>
<td>-0.042/-0.096</td>
</tr>
<tr>
<td>90</td>
<td>644.69 10^2</td>
<td>-107.40 10^3</td>
<td>-0.053/-0.105</td>
</tr>
</tbody>
</table>

σᵣ: stress at extrados.
σᵢ: stress at intrados.
0°: cross section at springing.
90°: cross section at crown.

The analysis of a tri-dimensional structure is reduced to the analysis of many mono-dimensional ones. Therefore, the parapet works like an helical beam, fixed at the foundation and hinged at the top, loaded by dead weight and part of flight weight. The remaining flight weight is borne by boundary walls.

Secondly, the flight is evaluated as tri-dimensional element. The structure is schematised as an helical beam, fixed at the foundation and at the top, which cross section amounts to masonry vault and parapet, forming a complex cross section. The fixed hinges along the boundary wall could be neglected because the bricks of each arch are tangent it. The analytical solution of this static scheme could be found in bibliography (Belluzzi O. 1994, Pozzati P. 1972); the difficulty consists to define some geometrical characteristics (torsion inertia moment).

These static schemes are the starting point for developing the research on this staircase typology.
Helical masonry vaulted staircase in Palladio and Vignola's architectures

The model complexity requires the employment of calculus tools as finite element method. The 3D modeling of Vignola's staircase is in progress.

Vignola's staircases are an unique masterpieces in the staircase architecture everywhere, which could not be catalogued within traditional technological and building definitions of handbooks.

CONCLUSIONS

Palladio e Vignola turned their attention to staircase theme, in private building, with different approaches. Concerning the helical masonry vaulted staircase, the following remarks could be done:

- Palladio's staircase offer a limited view due to the presence of central masonry pier, whereas Vignola's staircase is visible wholly with scenic views due to stair-well;
- both the architects enclosed their staircases in massive masonry rooms, choosing the best technology for them realisation;
- the structural analysis of these staircases has to be carried on considering the geometrical and technological aspects, as the historical one;
- their structural behaviour is difficult to be defined and a finite element model has to be done for verifying some hypotheses carried out during the research;
- their dynamic behaviour has to be investigated too. The Italian code imposes that the cantilever masonry staircases have to be demolished and rebuilt in reinforced concrete or steel (D.M. 02/07/81; D.M. 24/01/86; Lenza and Rampolla 1987); the restoration is possible only for historical and architectural value. Nevertheless, it is possible to preserve these structures defining their safety margin after a precise investigation about their geometry, technology and damage using calculus tools calibrated for the specific historical building (Barbieri, Foraboschi and Siviero, 1997).
- The author wish that the historical structures could be not replaced with new reinforced concrete or steel elements. Nowadays, it is possible to employ innovative materials, like
fiber reinforced polymers, able to supply tensile strength to masonry structures under ultimate loading condition.

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