The elliptical masonry dome of the Sanctuary of Vicoforte near Mondovì in northwest Italy, built in 1731, is the largest dome of this kind in the world (Figures 1–4). With its exceptional dimensions (major and minor internal axes 37.15 and 24.80 m, maximum height of the monument 84 m) it largely surpasses the dome of S. Andrea del Quirinale (1658) by Bernini (whose dimensions are 25.80, 16.25 and 25.80 m, respectively) and a group of other elliptical domes both in Rome (S. Carlo by Borromini, 1638, and S. Giacomo by Volterra, 1592) and in Spain (S. Hermanegildo in Córdoba, 1616 and Convento de las Bernardas in Alcalá, 1626) with major axes between 23 and 25 m and minor axes between 16 and 19 m (Zocca 1945, Escrig, Cobreros and Valcárcel 1997). Larger dimensions can be found only in the small group of famous axi-symmetrical domes comprising the circular domes of the Pantheon (2nd century, internal diameter 43.30 m) and San Pietro in Rome (16th century, 42.84 m), and the octagonal dome of Santa Maria del Fiore in Florence (15th century, 42.0 m diameter of the inscribed inner circle). The circular domes of Hagia Sophia in Istanbul (6th century) and of St. Paul in London (17th century) have an inside diameter of about 33 m.

These exceptional dimensions are to be related to the official role originally conceived for (but never assumed by) the monument intended to become the mausoleum of the Savoia family that governed Piedmont and southeast France since 15th century.
Unfortunately the stability of large masonry monuments, and of big domes in particular, is threatened by progressive fracture due to aging and chemical degradation of materials combined with the static and dynamic effects of dead load and ambient actions (thermal loads, wind, seism, traffic, etc.), settlement of foundations, yielding or delayed rupture of original iron reinforcements, etc. (Como 1998).

The exceptional dimensions of the Sanctuary of Vicoforte and especially of its dome, together with the peculiar geometrical shape of the dome itself and the particular slenderness of the drum-dome system (which are responsible of a complex and particularly severe static behaviour), and, last but not least, the unfortunate selection of the site of the Sanctuary from a geotechnical point of view, have largely increased the fragility of this monument, which has suffered significant structural damages during more than two centuries and was consequently submitted to an important intervention of structural strengthening two decades ago.

The principal structural damages consisted in:

- very large foundation settlements during the construction of the massive lower parts of the monument at the end of 16th century, and after the construction of the dome in the 18th century (maximum differential settlements of the west-side foundations with respect to the north-east side, developed during the whole history of the monument, were estimated in 1935–45 to be of the order of 33 cm) (Figure 5, Garro 1962),
- progressive opening of large meridional cracks reaching a maximum width at the base of the dome of 82 mm for a single crack, and a total amplitude along the perimeter of 413 mm (Figure 6, Garro 1962).

The project relating to monitoring, rehabilitation and structural strengthening was started in 1976 and (a) structural and geotechnical investigations of foundations and foundations layers, (b) rehabilitation of the drainage system of clay-silt layers, (c)
structural strengthening of the dome through the formation of a post-tensioning ring at the base of the dome, and (d) monitoring of the principal parameters characterizing the structural disorder and of the response of the monument after strengthening were carried out (Pizzetti and Fea 1988, Chiorino, Fea and Losana 1993).

A new five years program of monitoring, research and investigations —named Vicoforte 2002–2006— aimed to the control of the present conditions of structural conservation of the monument and to the establishment of the correct criteria for its future maintenance and, where needed, further structural restoration, was started in January 2002 under the coordination of the Politecnico di Torino and the responsibility of the second and third authors. As a part of this program a joint project of research has been established between the Politecnico di Torino and a group of Japanese Universities and research institutions under the coordination of the first author.

The objectives of the program Vicoforte 2002–2006 are the following:

1. Improvement of the geometrical and structural description of the monument
2. Diagnostic inspection of deterioration and structural damage by means of non-destructive tests
3. Characterization of the properties of the masonry by non-destructive and, where feasible, partially destructive tests
4. Investigation of the mechanical characteristics and continuity of the three original sets of annular iron ties embedded at the base of the dome
5. Geotechnical characterization of the foundation layers by field and laboratory tests
6. Installation of a new and extended monitoring system for the continuous control of the main parameters characterizing the behaviour of the monument, with special regard to the dome, and of the foundation layers
7. Measurements of natural frequencies and modes by ambient vibration and continuous registration of the dynamic responses to seismical events of selected magnitude
8. Interpretation of the static and dynamic behaviour of the dome and of the monument as a whole by means of finite element three-dimensional elasto-plastic and dynamic analyses, including its foundations layers, with inclusion of proper damage criteria
9. Updating of numerical models by comparison with experimental measures
10. Estimation of the global safety of the drum-dome system by limit analysis
11. Proposals for structural conservation and maintenance with special attention to the post-
tensioning ring at the base of the dome and to the definition of optimum stress levels for the steel tie-rods.

12. Analysis of the site of the monument from the hydrological point of view and estimation of the risks connected with extreme events.

The Italian team of research addresses in particular objectives 1, 5, 6, 10 and 12, while the Japanese team focuses mainly on objectives 2, 3 and 4, the remaining objectives are addressed as a joint project.

Preliminary results concerning (a) diagnostic, inspection and material characterization of the monument, and (b) estimation of global safety of the dome-drum system by limit analysis will be published in separate papers. The present paper addresses essentially point 8 and presents the preliminary stages and results of the research program concerning the numerical modelling of the entire monument.

A series of partial analyses by finite element method (FEM) limited to the dome alone were performed in the past years in connection with the rehabilitation works (Bernasconi and Marchini 1979), and subsequently in the frame of a research program at the Politecnico di Torino (Cecca 1994). The present work presents for the first time a FEM three-dimensional analysis of the entire construction (Yamaoka 2002). At this stage of the research program the mathematical model does not yet include the foundation layers, whose geotechnical characterization will result from step 5 above, and the structural characteristics of the masonry construction are investigated on the basis of linear-elastic analysis. Inclusion of foundation layers and consideration of damage criteria and influence of crack formation and propagation will be part of a further study.

By consequence the present study must be considered only as an initial exploration of the structural geometry and of the basic static pattern of the monument. Nevertheless, some preliminary indications on the critical aspects of its structural configuration can be obtained and they can help both in the definition of the further steps to be accomplished for the refinement of the numerical model, and in drawing the general lines for the program of monitoring and of experimental tests.

**BRIEF HISTORICAL NOTES**

Carlo Emanuele I of Savoia (1562–1630) decided to build a sanctuary, in the form of a basilica, on the site of a holy chapel dedicated to the virgin. The sanctuary should become the official mausoleum of the dynasty. The original project is due to Acanio Vitozzi (1539–1615) (Figure 7) and the construction was started in 1596. Due to an inadequate choice of the site (1/3rd on consistent marl in the north-east side and the remaining 2/3rd on compressible clay-silt layers of variable thickness up to 3.5 m) large differential settlements took place and the construction was practically abandoned at an elevation of 10 m at the end of 16th century, while continuing drainage works in the clay layers during the first part of 17th century.

In 1715 Vittorio Amedeo II di Savoia (1666–1732) decided to build the Sanctuary of Superga on the hills near Turin as the new mausoleum for the Savoia dynasty, and assigned the project to Filippo Juvarra. Although the representative role of Vicoforte was consequently fundamentally lost, architect Francesco Gallo (1672–1750) of Mondovi convinced the royal family to complete the construction of the basilica and built the baroque dome in 1731 after compensation of settlements at the base of the drum. Differential

![Figure 7](image-url)

Original project by Ascanio Vitozzi (1596)
settlements developed again due to the new weight of the dome, and cracking of the dome and the drum started developing and progressively increased until the recent application of the strengthening system in 1985-87. No substantial increases in the crack widths were observed afterwards.

**GEOMETRY OF THE DOME**

The horizontal section of the dome, normally referred to as elliptical, appears to be, after inspection of the photogrammetrical survey of the intrados of the dome, an oval consisting of 4 circular arches, very close to an ellipse. The curvatures vary moving from the base of the dome to the lantern, due to the different ratios of the diameters. The base internal diameters are 37,15 and 24,80 m (ratio 1,5) and the diameters at the base of the lantern are 7,74 and 5,94 m (ratio 1,3) (Figure 8).

The internal height from the base of the dome to the lantern is 16,66 m. The internal meridional curves that can be obtained from the photogrammetrical survey can best be approximated by arches of ellipses. It can be presumed that the control of the geometry during the construction was performed on the oval parallels, easy to be traced.

The extrados curve of such a baroque dome is more complex. The thickness of the dome varies from 1,91 m (1,70 m) above the windows to 1,37 m (1,17 m) in the proximity of the lantern for the transversal (longitudinal) cross sections (Figure 9).

**ORIGINAL CIRCULAR TIES AND RECENT STRENGTHENING**

The dome and the drum were originally strengthened by circular iron ties at three different levels, with a total cross section of about 140 cm² (Figure 10). A
program of non-destructive tests is presently under way to ascertain the mechanical integrity and characteristics of the original tie-system.

It may be interesting to observe that the cross section of the original ties for the dome of San Pietro inserted in 1588–90 by Della Porta at the moment of the construction (1588–90) reached only 60–70 cm², while the effective section of the additional strengthening rings applied by Poleni and Vanvitelli at the moment of the structural rehabilitation of the same dome (1743–48) reached 207 cm².

The modern strengthening system consists of 14 groups of tangential ties, each group consisting of 4 superimposed Dywidag 32 mm bars (32 cm² in total) of high-strength steel, normally used in prestressed concrete constructions, each bar being located in ducts drilled in the masonry (Figures 10 and 11). The 14 groups of bars are interconnected by steel trusses. The force in the tie-bars may be regulated at any time by jacks and the stress is constantly monitored by load cells.

**MONITORING**

The monitoring system includes, beside the load cells at the tie bars, a large number of displacement measuring devices (partly mechanical and partly electrical and automatic) applied to the main cracks, and temperature gauges. Measures of variation of
diameters have been performed manually until now. At present the monitoring system is under complete renovation and will include appropriate instruments for automatic measure of dimensional variations, crack openings, differential settlements, and accelerometers for measuring the response to natural (seism and wind) or artificially induced vibrations.

**NUMERICAL MODEL.**

As a first stage of analysis, the structural characteristics of the monument are investigated on the basis of linear-elastic analysis. The model, which is composed of 10-node tetrahedral solid elements, is shown in Figure 12. A second model composed of 20-node hexahedral solid elements is presently under construction and will be used for the second phase of the analysis in the non-linear domain. Cosmos 2.5 is used in this analysis. Total number of 10-node tetrahedral solid elements and nodes are 32,181 and 16,716, respectively. Based on experiments (Barosso 1979, Rodio SpA 1983), material constants used in the finite element analysis are determined as follows: Young's modulus 15,000 kgf/cm², Poisson's ratio 0.15, weight per unit volume of the masonry 1.700 kgf/m³, compressive strength 30 kgf/cm², tensile strength 3 kgf/cm².

**PRELIMINARY RESULTS OF THE ANALYSIS**

The result of the analysis in case of dead load is shown graphically in Figures 13 to 15 representing the state of deformation. The ellipse delineated by the
Figure 14
Deformation in north-south direction

Figure 15
Deformation in east-west direction
base of the dome deforms into a more acute-angled ellipse. The massive buttress piers on the north and the south sides deflect outward due to the thrust of the dome and diagonal crack develop, as can be seen in Figure 14. On the eastern and the western sides, however, as shown in Figure 15, the upper parts of the massive piers deflect inward due to the eccentricity of the given load against the lower supporting structure. This oval displacement pattern of the base of the dome is in accordance with the actual state of the structure.

Principal stress $\mathbf{1}$ is shown as a vector field in Figure 16. The vectors with dark colour represent large tensile stresses. The crack of masonry is assumed to occur when the tensile principal stress exceeds the tensile ultimate strength. The direction of the crack is perpendicular to the direction of the tensile principal stress. According to the direction of the principal tensile stress, the vertical cracks of the dome and drum correspond well to the computed state of stress of the whole structure (Figures 6 and 17).

**CONCLUDING REMARKS**

Though the actual deformation is too large to be analysed in the elastic phase of the material, our results correspond well to the oval deformation observed along the base of the dome. They show also that the locations of both the original annular iron ties, of inadequate section, and of the post-tensioning belt embedded at the base of the dome in 1987 were properly selected.

The results of non-destructive tests will be used for updating the numerical model. A second phase of the study will concern elasto-plastic analysis with inclusion of proper damage criteria for the material.

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