Inventing a history for structural engineering design

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It is often convenient to assume that history is a series of facts waiting to be discovered. But there are so many facts. The facts that we do discover are, to a large extent, auto-selected according to who we are and why we are looking for them. This influences where we look for facts and what we do with them when we have found them. The history that is so generated can be very personal, even subjective, and the great challenge is to persuade others of the value or reward to be gained from adopting a certain approach to history.

As a practising design engineer, the author’s own journey into the past began with questions about how people in former times might have designed things. The search uncovered a wonderful range of material in various different strands of structural engineering history:

— the development of different types of structure (e.g. suspension bridges, timber roof trusses, dams, masonry domes);
— developments and inventions in engineering technology (especially materials, plant and machinery, and methods of manufacture or construction);
— biographies of engineers;
— the stories of individual projects;
— developments in engineering science and mathematics («theory» and techniques of analysis), and
— various non-technical issues (economic, commercial, contractual, management, social, political, etc.).

Despite much searching, however, it became clear that very few people had attempted to answer the author’s most burning question — how were the structures designed? To a large extent, the challenge became to create or invent the idea of a history for engineering design. Rather than using someone else’s map to navigate the territory, the challenge was to draw a new map. And an important aspect of this challenge was to distil the very essence of engineering design, and, especially, to distinguish it from general ideas of «technology» and «engineering science».

Engineers themselves have seldom been effective in promoting the nature of their art — they have generally been too busy getting on with projects — and this is nearly as true today as five hundred or a thousand years ago. Professor Fritz Leonhardt, from Stuttgart, was one of the most eminent design engineers of the 20th century. In a book about what engineers do he wrote:

In essence, to design an artefact is to plan what you are intending to do before you start, and to devise a practical and economic way of constructing it. With characteristic simplicity, Ove Arup once wrote that:

Design is nothing else than indicating a sensible way of building. It includes all drawings, specifications, descriptions and detailed instructions about what should be built and how it should be built (Arup 1985).

Even definitions such as this, however, overlook a most important aspect of engineering design—the process of achieving the necessary confidence to begin building. It is one thing to sketch a magnificent bridge or roof for a building, quite another to be confident that it can be constructed and will safely carry the loads to which it may be subjected. This aspect distinguishes fundamentally the work of those engaged in civil and structural engineering from those engaged in the design of small machines or consumer artefacts, who never need to address questions as to whether their creation will work first time, or be safe, or be possible to make.

Engineering design is a process with two key outputs:

— a description of what has to be manufactured and built: materials, specifications, dimensions, construction method. This may be a verbal instruction, but, especially when the building is large or complex, would usually be written down or in the form of a drawing or a model; and

— a justification or explanation of the design proposals that have been made. This may be achieved on the strength of precedent, full-scale tests, experiments on small physical models, various calculations, or the use of mathematical models, nowadays based on engineering science.

A study of what engineers do can make the process seem all very mechanical, with resulting artefact an almost inevitable result. But this is not how it seems to the design engineer, him or herself. It is important to consider the challenge and the unknown aspects of a project that the design team faces. From an engineer’s point of view, the principal achievement in constructing the Pantheon, or a large cathedral or the Eiffel Tower, was that some people were able to convince themselves that the resulting structure could, indeed, be built, and then that they were able to persuade the clients and even the public that they would be possible. And furthermore, they were so successful in putting their case that they were able to raise the considerable sums of money necessary to undertake the projects.

If the engineer’s job is to be summed up in one phrase, then, it can be to create the confidence to start building. This provides a focus for historical study. How, at different times and places, have engineers found the necessary confidence to build?

If, at any time in history, the project engineer did not feel confident that a certain structure would work, he would have to decide what he needed to do in order to gain that confidence. This is the art of the engineer. Seeing and understanding how something similar has already been done is probably the most important help an engineer can get. For unprecedented or especially challenging problems, scientists and mathematicians (often called «philosophers» in ancient times) have also been an important source of expertise that can raise the engineer’s confidence.

This confidence must overcome any doubts about success the engineer may harbour, as well, of course, as any doubts by the client whose money and reputation are also at risk. A history of engineering design, then, must address these different aspects of the design process:

— what was the final design for the artefact or structure?
— how was the final design arrived at or devised?
— what means did the design engineer use to increase his confidence, before commencing construction, that the proposed structure would work and could be constructed, sufficiently for him to begin building.

A history of confidence among engineering designers would be a challenging subject to undertake since confidence depends very much on the experience and the cultural and intellectual background of the engineer as well as the context in which the engineer is working. There are also important questions about how different engineers in the past might have perceived the gap between what they knew had already been done and what they
might have believed to be possible, though not yet know by what means. And the confidence is needed at even the most fundamental levels such as the reliability and sophistication of mathematical calculations and the accuracy of dimensions generated by geometric techniques of producing drawings of complex shapes or connections in a building. A history of engineering design needs to address these aspects of the skill. This was a challenge that faced the author when working in the team devising an exhibition about the work of engineers some years ago—had do you put on display? On that occasion the answer to this question included a collection of the engineer’s aids to calculation—tables of logarithms, slide rules and mechanical and electrical calculators. There were also some design engineers’ notebooks with their little sketches and the calculations they made when exploring a new idea. These things really do capture something of what engineers do, but their apparent insignificance may still give only a hint at how a final grand scheme may have come about.

There are, of course, problems in using modern words like «engineer», «design» and «designer» to refer to the construction of a Gothic cathedral or, indeed, any edifice completed before the twentieth century when these words acquired their modern meanings. The author has not found it difficult to use such words to describe the past, without implying the professional demarcations, knowledge and working methods of the twentieth century. At the risk of being accused of tautology, it is possible to use them to embrace the work that must have been undertaken to complete a cathedral, without specifying precisely what it was or who did it (since we do not know). It is possible to discuss the process of designing a cathedral, for instance, without becoming entangled in questions as to whether it was designed by an architect or an engineer, or whether its structure was designed separately from the architecture. These questions are, literally, meaningless because to use the very words «engineer» or «structure» is anachronistic.

What happened, over a period of many centuries, was that the activity of designing a large building, whoever did it, was gradually broken down into more and more distinct issues. Thus the methods of designing a Greek temple or a cathedral embraced visual appearance, sense of space, function, materials and structure, as well as what we now call the internal environment (lighting, heating, ventilation and acoustics) in a single, holistic design process. The visual appearance, sense of space and function (the «architecture» in the modern, narrow sense) became a distinct concern during the fifteenth and sixteenth centuries. About a century later, designers first began to think about the load-bearing aspects of buildings in terms of loads (weight), materials and structure. Thinking separately about materials and structures grew during the late-seventeenth and eighteenth centuries, following Galileo’s work, and aspects of the internal environment came to be considered independently of the fabric of the building during the late-nineteenth and early twentieth centuries.

It is also worth noting that structural understanding is neither a new phenomenon, nor one that requires a knowledge of statics and elasticity. It is a commonly-held misconception that new types of structure and structural forms were devised first by mathematicians or scientists and later taken up by builders, engineers or other designers of structures. In fact, the opposite is the case, with perhaps just one exception—the hyperbolic paraboloid, whose structural properties were discovered in the 1930s. Many children and sculptors display a remarkable understanding of materials and structures without a knowledge of engineering statics, and many structures from long ago show their creators understood the essence of all the basic structural actions better, perhaps, than many engineers today.

A history of structural design would need to address how all the structural elements of buildings were designed. A few examples do exist—for example, beams (Yeomans 1987; Skempton 1956, Sutherland 19), foundations (Peck 1948), retaining walls (Kerisel 1993), and stability design (Mainstone 1988).5

Two examples from the history of engineering design will illustrate some of the points made above—the design of Gothic cathedrals and the design of beams for use in buildings.

THE DESIGN OF GOTHIC CATHEDRALS

In some ways, designing a cathedral is not as difficult as it may sound. Most importantly, we already know that it can be done—and this was true in 1200 too, for
many large cathedrals had been built before the period we know as Gothic, especially in France. Nowadays we also know that the masonry is loaded only in compression and that the stresses in the materials are very small compared to the strength of the stone or brick (less than about 10%). The strength of the masonry is, therefore, virtually irrelevant to the success of the structure as a whole.

The structure must, however, be stable. Arches and vaults must not thrust sideways so much that they push over the walls or columns; they must also be curved such that lines of thrust remain within the thickness of the masonry. Also, the loaded elements must not be so slender that they fail by buckling. The stability of a masonry structure depends, then, on the relative size and disposition of the individual pieces. It does not, generally, depend on the absolute size of the building. Hence it is easily possible to build a model masonry structure and scale it up to whatever size you want—the equilibrium conditions for the model and the full-size structure are the same. Nowadays we can demonstrate this using statics; engineers also knew this in 1200, based on their own direct experience and the evidence of thousands of successful masonry buildings.

The exception to these generalities about scaling up from models is the question of wind loads. A masonry structure must not be overturned by winds from any direction. We know that the force exerted by the wind on an object does not vary in simple proportion to the wind speed. We also know that the speed of real winds increases as you rise further above the ground. It is quite likely that these non-linear effects were the cause of some collapses of early large masonry structures, especially during construction. The main causes of damage to cathedrals were, and still are, in fact, the settlement or movement of the foundations and the burning of roof trusses after lightning strikes.

Something dramatic happened to cathedral and church design in the 12th century. After several hundred years of gradual development since the end of the Roman empire, the designs for large churches took a sudden and sharp change of direction. Round arches were replaced by pointed arches, long barrel vaults were replaced by several discrete structural bays formed by quadripartite vaults (intersecting pointed vaults), the groin vault was replaced by the ribbed vault, columns were reduced in thickness, the maximum area of window in a wall increased from perhaps 30% to around 80%, the horizontal thrusts of the vaults (and wind loads on the roof) were carried out over covered aisles by means of highly efficient flying buttresses. The world had seen no similar transition in building engineering since the 2nd century AD and would have to wait for 700 years for a similar period of dramatic structural development in early Victorian England. During the 400 or so years of the Gothic period there was some steady development and improvement in various elements of the cathedral, especially the slenderness of columns and flying buttresses. However, these changes were relatively small compared to the very sudden developments of the mid-12th century.

The beginning of what we know as the Gothic period can be pinpointed quite precisely as about 1134: in or around that year new building commenced on the cathedrals of St. Denis and Chartres. Within a few years several others were begun, all in the region around Paris called the Ile de France and by the year 1300, some 60 Gothic cathedrals in France and 40 in England, were complete or under construction. The style continued to spread throughout Germany, Italy, Spain and central Europe until the late 16th century.

By and large, we know how the cathedrals were built and we also know quite a lot about how they were designed. We know the names of many hundreds of the individuals who were involved with their design and construction. Unfortunately we have no explicit design procedures dating from the early Gothic period. This is hardly surprising considering the secrecy surrounding the skills of trades such as the mason—it was forbidden to divulge any information outside the masons’ lodge, either to other masons or non-masons. We also have direct evidence of what was designed—the buildings themselves. Many documents relating directly to the designs of cathedrals have survived including several sketchbooks of details and plans of cathedrals, the most famous of which is by Villard de Honnecourt. Many more substantial working drawings survive and we know that some of these formed part of the building contracts.

From such evidence it is possible to work out how some of the plans and shapes have been constructed. They seem to be based mainly on two geometric techniques or manipulations - the combining of various circular arcs and the «rotating of the square»
—a procedure by which a square with an area of half of another can be created by joining the centres of each side. These were used to generate an enormous range of plans and elevations for cathedrals and their components, and several notebooks from the late Gothic period include details of such geometric design procedures. We do have some examples of such methods, although from rather later, contained in some design from the 14th and 15th centuries (Shelby 1979; Sanabria 1982; Addis 1990).

At first sight, such geometric procedures would seem to be of little relevance to what we would now call the structural design of the building. Certainly they were not based on statics. But it all depends on how we look at the matter of design. If we are looking at the history of engineering design at a time when it is meaningless to talk of statics, any design procedures must be of interest, especially in the light of the discussion about the nature of design earlier in this paper. It is also worth looking more closely at what was meant by «geometry» in the 12th and 13th centuries (von Simson 1952, 1956).

Most importantly we know that, somehow, building designers were inspired with a new confidence to push back the boundaries of what was possible and to be much more economical in their use of materials. How did people come to believe that it would be possible to build cathedrals taller, wider, longer and more daring than ever before? What happened, or might have happened to give rise to this confidence? Part of the answer seems to be bound up with geometry and, in particular, with Euclid.

The role of geometry in the mediaeval world

Since Classical Greek times (and probably earlier), people had sought explanations for natural phenomena in order to understand the world they lived in — rainbows, the musical notes made by vibrating strings, the orbits of the stars and planets, the trajectory of missiles, and so on. In every field of what we now call natural science — optics, acoustics, astronomy, music, mechanics, botany — philosophers had explained phenomena in terms of geometry, harmonics and number which were seen as earthly manifestations of the principles the god(s) had used to create the universe. These explanations were generally expressed in terms of the absolute truths of number and geometry. With numbers there were ratios, squares, multiples, series and so on; in geometry there were circles, triangles, squares, spheres, cubes and so on, and the properties associated with these shapes.

This use of geometry, harmonics and number was also incorporated into Christian philosophy. Saint Augustine (354–430) and Boethius (480–525) (both were philosophers and hermeneuts) provided the means. Their works on the science of music, mathematics and architecture sought to demonstrate, using harmony and geometry, the underlying principles of the world as created by God. Further manifestations of these principles were, of course, to be found in the Bible. Augustine took the Biblical passage Omnia in mensura et numero et pondere disposeris (Thou hast ordered all things in measure, number and weight) and applied Pythagorean and neo-Platonic methods to the interpretation of the Christian universe, its creation and its order. Concerning the design of buildings, the Bible also provided dimensional details of a number of significant structures — the Ark, Moses’ Tabernacle, Solomon’s Temple and the Celestial Temple revealed to Ezekiel in a vision. A well-known 14th century masonic poem even claims that Solomon actually «taught» architecture in a manner «but little different from that used today» and that this science was directly transmitted to France. The writings of Augustine and Boethius dominated the middle ages, and the cosmic applicability of the laws of harmony figures boldly in writings about both music and building throughout the Gothic period.

It was into this philosophical tradition, in northern mediaeval France, that the books on geometry by the Greek mathematician Euclid were suddenly introduced. All copies in Greek or Latin had been lost or destroyed and the books had survived only in Arabic translation. While the English scholar Adelard of Bath was a student of Thierry at Chartres, he visited Domenicus Gundissalinus, the Archdeacon of Segovia, which had recently been recaptured from the Moors. There he came across a copy of Euclid’s _Elements of Geometry_ in Arabic which he translated into Latin and brought back to his community of fellow scholars at Chartres in the mid 1120s (some say a few years later). The appearance of Euclid seems to have had a profound effect on their work — it has been said that, under this influence from Euclid
«the school of Chartres attempted to transform theology into geometry».

Geometry had, of course, survived as a practical art throughout the Middle Ages and the appearance of Euclid improved the level of geometrical knowledge which could be learnt. Improved geometry facilitated the more accurate description of proposed building designs and was of great practical use in the construction process —setting out buildings and enabling the accuracy of the finished parts and their relative disposition to be checked to greater accuracy. Such an improvement alone would have enabled builders to contemplate larger and taller buildings.

However, it was in the capacity to provide justification of designs that geometry probably had a more profound effect. Euclid introduced a crucial new ingredient —the notion of the geometrical proof. This provided the perfect tool for the philosophers at Chartres to argue their views logically and to justify decisions made in a wide variety of contexts including, probably, the design of buildings. Just as occurred after the invention of calculus some 600 years later, philosophers put the new theoretical tool to use in every conceivable way and created, quite literally, a new type of geometry — geometria theorica.

The distinction between geometria theorica et practica was probably first made by Hugh of Saint Victor sometime between 1125 and 1141. In doing so he was looking back to the philosophies of Plato and Aristotle in distinguishing the practical skills from the theoretical (contemplative) skills which had been made possible by the appearance of Euclid. Hugh put the theoretical tool to good use in helping to explain and justify information given in the Scriptures. For instance, he calculated that the reported size of the Ark (40,000 inches) would indeed have been large enough to accommodate all the animals of the world and their food (see Victor 1979, 3 and 32).

There soon followed a number of geometry text books, some purely practical, and others, such as one written around 1140 by Gundissalinus which also dealt with theoretical matters. In this work he distinguished the two geometries according to their respective purposes and duties (Table 1).

<table>
<thead>
<tr>
<th>Geometria theorica</th>
<th>Geometria practica</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finis (purpose)</td>
<td>to teach something</td>
</tr>
<tr>
<td></td>
<td>to do something</td>
</tr>
<tr>
<td>Officium (duty)</td>
<td>to give reasons and dispel doubt</td>
</tr>
<tr>
<td></td>
<td>to give measurements or limits which the work should not surpass</td>
</tr>
</tbody>
</table>

Table 1
The several functions of mediaeval geometry, according to Gundissalinus cited in (Victor 1979: 9)

Other 12th century geometry treatises appeared. The prologue to one of these:

...almost makes the practical side of geometry seem subservient or secondary to the theoretical. The use of theoretical methods in practical geometry seems to have increased between the twelfth and the fourteenth centuries. At first their role was ancillary to the purposes of practical geometry. Once proofs had found a place in practical geometry, their role increased and changed. Theoretical proof became the goal even of practical geometry. (Victor 1979)

We also find, in the work of the philosopher Robert Grosseteste (c. 1175–1253), just a few decades later, perhaps the first mention of what is now called the hypothetico-deductive method in science, and the principle of falsification whereby a hypothesis should be rejected if it leads to conclusions found to be at variance with experience. Grosseteste typified the contemporary attitude to geometry in declaring that:

*without geometry it is impossible to understand nature, since all forms of natural bodies are in essence geometrical and can be reduced to lines, angles and regular figures.* (Victor 1979)

In brief, we find the role of geometry in the mediaeval period was rather like the role that modern science —physics, mechanics, materials science, etc.— plays in the world today. It provided explanations of why the world and the heavens were as they were, and how they worked.
The gothic design revolution

To a medieval cathedral designer, then, a geometrical model of the building could serve not only as a mathematical model to describe the proposed construction, but also, being based on geometry, it would serve in some manner to justify its adequacy, in conjunction with other engineering knowledge, of course. While we do not know exactly how they used geometry to «give reasons and dispel doubts», the explosion of interest in geometry during the 1120s and 1130s did occur at the very beginning of what we now know as the Gothic era. The cathedrals of St Denis, Sens and Chartres were all designed in the 1130s; another dozen followed during the next 25 years.

Any plausible account of building design in the Gothic era must take account of the significance of geometry in the mind of medieval man. Even the scant evidence presented here would indicate that geometry would indeed have helped provide the early cathedral designers with the confidence they needed to propose bolder and bolder designs for the cathedrals.

The changes that occurred in the design of cathedrals during the 12th and 13th century were not technological developments in the conventional sense — no new materials or structural devices were invented. The changes were largely human changes—they went on in the minds of the designers and builders. For this reason it is appropriate to refer to this sort of change as a revolution—a design revolution.

These changes were largely complete by the middle of the 13th century and comprised the first of two design revolutions during the Gothic era. The other began about a century later when the first signs began to appear of taking into account the weights of materials and the loads these imposed on parts of buildings (Sanabria 1982; Addis 1990).

The design of beams

The second story in the history of structural engineering took place over many centuries—the development of design methods for beams used in structural frames. One key moment in this story was the development of the I-beam and its introduction into buildings in the 1830s.

Beams have been used in buildings for thousands of years for floors and roofs. In classical Greece the choice was generally between timber and stone. Both of these materials occur in nature in larger sizes than they are needed in buildings so extra work is needed to reduce the size down to what would be the minimum possible. It was therefore normal to make a compromise and use a form for beams that was easy to manufacture and large enough to carry the loads. Most beams were, therefore, of rectangular cross section and constant shape along their length. It was well-known that the strength of a beam increased in direct proportion to its breadth and more than in direct proportion with its depth (with the square of the depth, in fact).

To conceive most buildings, therefore, the choice of size was very limited and easy to learn and pass on to young builders. It would depend on whether:

- the beam was in the floor or roof (i.e. on the load)
- the beam was of wood or stone
- the type of wood (or stone)
- the span between the supports

The types of material and the dimensions needed were well known and would have hardly needed to be written down. Nevertheless, as our modern scientific approach to the world developed during the 18th century manuals did start to appear, especially for timber. These gave tables and simple formulae for establishing the scantlings (dimensions) of the timber components of floor and roof structures (Yeomans 1987). There was little or no attempt to justify these dimensions, for they were so well-established.

In fact, there are some very early examples of stone beams from the 5th—3rd century BC in which the cross section of stone has been reduced to less than a rectangle, in order to improve the structural efficiency (Coulton 1977). For the very largest spans—the one in Figure 3, from 400 BC, was 6.2 metres in length—the depth of the beam could be reduced away from the centre of the beam without reducing the strength too much. In modern language, more stone is used where the bending moment is larger and to improve the ratio of cross-sectional area to second moment of area. In this case, however, the increased efficiency of the beam was at considerable expense in terms of labour and would only be worth considering for unusually large spans.
As builder’s mathematical skills improved towards the end of the 18th century, some books contained simple formulae which effectively summarised the data previously presented in tables. These were empirical formulae and were not engineering science as we know it—they dealt with only one material and rectangular cross-sections, constant along the length of the beam. The enabled someone to calculate the relative strength of a beam-relative, that is to a similar beam but of smaller or larger dimensions. The formulae contained empirical constants with no scientific significance at all. The relative strength was distinguished from the absolute strength which was the concern of scientists seeking to explain how strong a beam would be.

This situation changed with the introduction of cast iron in buildings, as a fire-proof material for columns and floor beams at the end of the 18th century (Skempton 1956; Sutherland 1984). Unlike with timber and stone, when you make a beam from iron, it was important to use as little material as possible—the more material you use the more it will cost and you had to keep the structure as light as possible, since iron is three times more dense than stone and seven times denser than timber. There was, therefore, for the first time in history, benefit in using the minimum amount of material possible—the search for «minimum-weight» structures had begun and the established design procedures were unable to help designers in their task—a period of crisis had arrived. This started the urgent search for the most efficient cross-section of beam and shape, from end to end. In fact, the search did not take very long as much of the theoretical and practical work had already been completed by many scientists and mathematicians earlier in the 18th century.

The early application of simple bending theory did not, however, fully reflect the asymmetric properties of cast iron which is much weaker in tension than in compression. By the 1820s another moment of crisis in design procedures had arisen. This led William Fairbairn, with Eaton Hodgkinson, an engineering scientist at Manchester University, to search for new design procedures that would generate the optimal shape for a cast iron beam for use in the many high-rise factory buildings that Fairbairn’s firm was building. In this way he hoped to outdo the competitors! And he did. The result of the work was the I-beam (Hodgkinson 1831). However, this was not the symmetrical I-beam we now know, since cast iron is six times stronger in compression than in tension. The lower (tension) flange of the beam must therefore have an area six times that of the upper (compression) flange. The resulting section was first used in 1834 in Orelons Mill in Manchester and the earliest surviving example is at the famous Saltaire Mill by William Fairbairn (Fitzgerald 1988). This paper has reached only the middle of the 19th century in this extract from the story of designing beams, but it continues through the next 150 years right up to the present. Now the mathematical models of beams used by engineers to justify their design decisions are much more sophisticated and can even represent the behaviour of beams under fire loads as well as gravity loads, but that is another story.

**CONCLUSION**

This paper has sought to demonstrate that the historical development of engineering design is a subject both different and separate from other themes in the historical study in construction. At present the written history of engineering design is unevenly covered, even though the source material may exist. As yet, for instance, no-one has traced the history of how timber and, later, wrought-iron roof trusses were designed (as opposed to constructed). Rather more serious is the absence of due attention from historians to probably the two most important...
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subjects in the history of engineering design—the development and use by designers of graphical statics and the factor of safety. These are themes in need of research.

Unfortunately practising engineers often have little time or inclination to follow these directions, so it must be hoped that some professional researchers have been stimulated by this paper to take up the challenge.

Notes

2. For further discussion of these issues, see Addis 1990, 1994, 1997 and 2001.
3. The profession of building engineer is hardly known by the public. When building is mentioned, people usually think of the architect; when a building engineer designs something, the press refer to him as an architect. This has been my experience during my whole life. I am of the opinion that the building engineer himself is largely to blame for this state of affairs.
5. These and other examples are reprinted in Addis 1999.
6. These issues are discussed in depth in Addis 1990. This book is now out of print, but a few copies are available from the author.
7. This phrase is drawn, by analogy, from the idea of the scientific revolution that the American philosopher Thomas Kuhn developed some thirty years ago and which has transformed our understanding of the development and history of science (Kuhn 1970, Addis 1990).
8. These two papers are included in Addis 1999.
9. This paper is included in Sutherland 1997, along with Sutherland 1990.
10. A further episode in this story, the design of steel beams using plastic theory, is given in the author’s other paper in this conference: The nature of Progress in structural engineering.

REFERENCE LIST


